Meditations on Breguet and Mathematics

Richard Watkins

In the second edition of *"The Origins of Self-Winding Watches"* (Watkins, 2016) I discuss two types of stop-work used by Breguet in his self-winding watches. I also describe the planetary gears used in rotor watches. Both descriptions hint at, but ignore some interesting features of these gears. This article expands the explanations by looking at these largely irrelevant features.

1: God and Relativity

We all know that, *relative to ourselves*, the Sun rotates around the Earth. From our perspective we are stationary and all things move around us, the Sun, the Moon, the stars, etc.

But we are taught that this is not true and, in fact, we are moving around the Sun. Indeed, the Moon rotates around us, we rotate around the Sun, the Sun rotates around the galaxy, ...

However, in practice this knowledge is largely useless because we are not in a position to experience this behaviour. Except for jet planes. Sit in a plane and look out a window. *Relative* to us and the plane we are stationary and the Earth rotates below us. Sit on the Earth and look up. *Relative* to us and the Earth we are stationary and the plane moves above us.

Although we might be able to understand the "universe" by observations and mathematics, we can only experience it from within. Thus there is an interesting difference. We are within the universe, but we are without the watch, and so our relative perspectives are dramatically different. Until now I have never seen watches examined from *within* the watches ...

For God's first attempt to create a universe he copied the idea from another deity; it was remarkably similar to the one we live in. However, he realised he had made a few mistakes. The year was about 365.25 days and the week had 7 days (because it took him that long to create it), so calculating anything was a nightmare. The inhabitants were forced to invent leap years, have months of different lengths and create the calculus to understand it all. However, because the universe was irrational there was considerable religious fervour, which was pleasing.

Before trying again, God decided to learn arithmetic so that he could do a better job. He also read Paley's book (well, the first few pages as the rest is too tedious). The result was that God decided to base his next universe on watches. However, he did not really understand watch trains, so he used two meshing wheels, Figure 1.

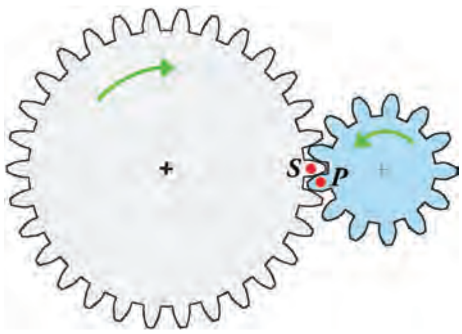

Figure 1

The wheel ***S*** has 30 teeth and it rotates once every year. There are 360 days in a year and 6 days in a week (because it took 5 days to make this flat universe and, as was standard practice amongst the deities, God took off a day to admire his workmanship). Thus there are 60 weeks in a year and each tooth represented a week (the 30 spaces between the teeth are also 6 days). And there are 12 months, all having 30 days.

The pinion ***P*** has 13 teeth. This came about because God had some trouble using a ruler, compass and protractor and got it wrong. But it turned out to be a good idea. The inhabitants had very little to do other than some simple multiplication and division of integers. So the behaviour of ***P*** gave them much to think about.

These wheels cannot move sideways because God mounted them on pivots running in holes in space, and they can only rotate around their centers. Because God knew a little about watch trains, he knew the relationship between them:

$Tp = -Ts\ (Ns/Np)$

That is, if ***S*** turns once clockwise, $Ts = 1$, the pinion will turn 30/13 times anti-clockwise, the ratio of the number of teeth of *Ns* and *Np*. The minus sign tells us that the direction is reversed.

Actually, the terms clockwise and anti-clockwise are irrelevant, because whatever direction ***S*** turns (and so *Ts* is positive or negative) ***P*** will turn in the opposite direction and *Tp* will have the opposite sign.

This relationship can also be expressed in terms of the radiuses *R* or the circumferences *C*:

$Tp = -Ts\ (Rs/Rp)$ or $Tp = -Ts\ (Cs/Cp)$

It is useful to remember that, because the meshing teeth must be the same size:

$Ns/Rs = Np/Rp$

Figure 2 shows this universe, as God sees it, 4 weeks later. But what the inhabitants see is completely different.

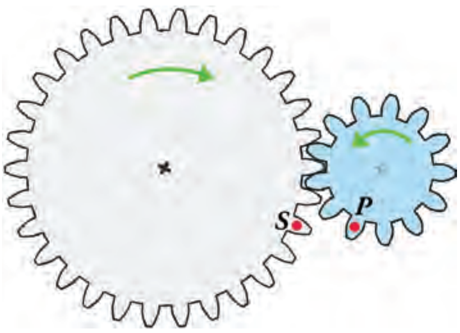

Figure 2

In Figure 1, Richard lives in a town on ***S*** at the red dot. And Joseph lives in a town on ***P*** at the other red dot, to the south of Richard. While the universe is in this position they can meet.

Relative to Richard ***S*** does not move and east is always to the right, looking out over the tooth (just as on Earth, where your house always faces in the same direction and east is a relative direction that has nothing to do with the position of Earth in space). Because ***S*** does not rotate, Richard defines a year as the time interval from Figure 1, when ***P*** is due east, until the next time ***P*** is due east. Fortunately this is the same as one God year.

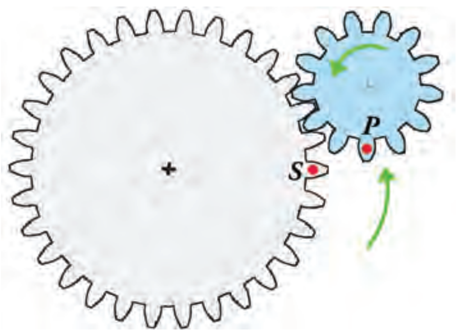

Figure 3

From *Richard's* perspective, sitting on ***S***, ***P*** both rotates and moves anti-clockwise in space and, when God's view is Figure 2, Richard sees Figure 3. ***P*** moves slowly to the north and, near the end of the year, reappears in the south. After exactly one year, Figure 4, ***P*** has pirouetted around ***S*** and is again to the east of Richard. But Joseph and his town are no-where to be seen!

Two years later, Figure 5, Richard met Joseph again, but this time he was to the north of Richard! Richard assumed Joseph must have moved to a new house and Joseph thought Richard had moved. Then Joseph disappeared and it is not until after a total of 13 years, Figure 6, that Joseph reappears to the south of Richard, in exactly the same position as in Figure 1.

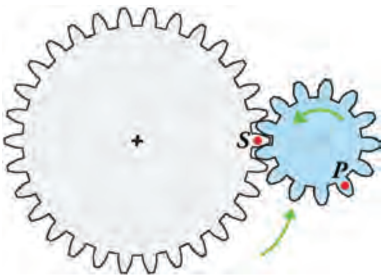

Figure 4

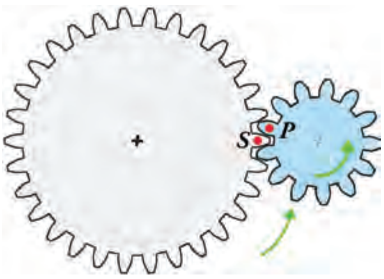

Figure 5

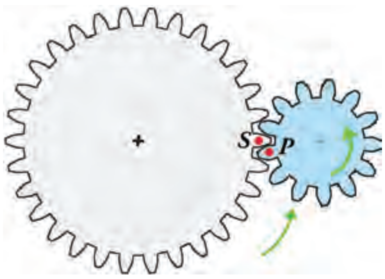

Figure 6

This bizarre behaviour (from the point of view of Richard) produced a number of doomsday prophets and caused consternation among the mathematicians. But after enough years they worked out the structure of the universe and discovered that:

$$Gp = remainder\ (30Ys/13)$$

That is: The gaps between the teeth of ***P***, counting clockwise, were numbered from 0 to 12, and the tooth occupied by Joseph is between gaps 0 and 12. Then, after Ys complete years Gp is the number of the gap opposite Richard; the sequence is 0, 4. 8, 12, 3, 7, 11, 2, 6, 10, 1, 5, 9, 0 … . Because space 12 is beside Joseph (on the other side) he could be seen after 3 years. (Of course, $Ys = Ts$, the number of turns of ***S***.)

From *Joseph's* point of view ***P*** does not move and ***S*** rotates. At the same time as God sees Figure 2 and Richard sees Figure 3, Joseph sees Figure 7, where ***P*** is stationary and ***S*** both rotates and moves clockwise around ***P***. So Joseph calculated his year as the interval between the times when a tooth of ***S*** appeared due west. Going back to God's view: $Ts = -Tp\ (Np/Ns)$.

That is a ***P***-year, $Tp = 1$, is $^{13}/_{30}$th of a year on ***S***, or 26 ***S***-weeks.

Because $Np = 13$, a prime number, the residents of ***P*** had to create an irrational system of time measurement, of no interest to us. But Joseph was interested to know when he would see Richard. Not surprisingly the tooth of ***P*** opposite ***S*** is:

$$Tp = remainder\ (13Yp/30)$$

That is, the years on ***P*** go in a 30 year cycle and Joseph sees Richard twice in this cycle, in years 0 and 23.

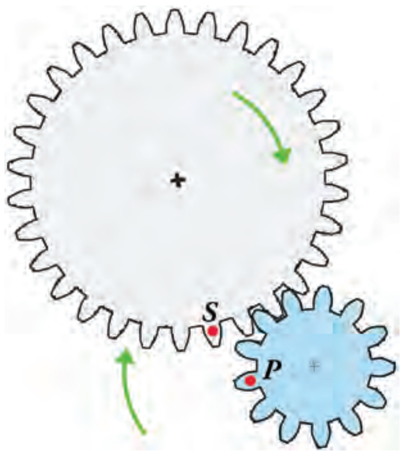

Figure 7

These three views of the universe respectively place space (that the wheels are mounted on), the center of ***S*** and the center of ***P*** at the center of the universe, with everything revolving around these centers.

Also, the concept of a year is meaningless, because it depends on the speed of rotation of the wheels. To avoid excessive bills from psychiatrists, God kept the direction and rate of rotation constant.

Rather pleased with his success, God decided to add a third wheel with 20 teeth and, with a flash of lightning and a clap of thunder, he transformed the universe from Figure 1 into Figure 8.

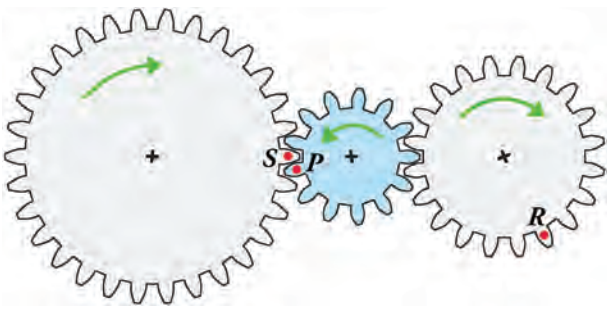

Figure 8

Much to his surprise, the red dot town on the new wheel ***R***, where Marco lives, was nowhere near anything. (Clearly God did not position that red dot, which raises a serious philosophical question about divine intervention.)

Anyway, the mathematics was easy. Considering ***P*** and ***R*** we have:

$$Tr = -Tp\ (Np/Nr)$$

And so:

$$Tr = Ts\ (Ns/Np)\ (Np/Nr) = Ts\ (Ns/Nr) = Ts\ (^{30}/_{13})\ (^{13}/_{20}) = {^3/_2}\ Ts$$

P is now acting as an idler wheel whose only effect on ***S*** and ***R*** is to reverse the motion of ***R***; without it *Tr = - Ts (Ns/Nr)*. And we (and God) can see that one year on ***R*** is 20/13 ***P***-years and 20/30 ***S***-years. Actually the relationship between ***R*** and ***P*** is exactly the same as the relationship between ***S*** and ***P*** except that the number of teeth have changed. From the point of view of Marco there is a 13 ***R***-year cycle for Marco meeting Joseph and a 20 ***P***-year cycle for Joseph meeting Marco.

Much more interesting is that at the time of the creation of ***R*** Joseph did not know it existed, because he was talking to Richard (Figure 8) and he knew nothing about what had happened on the other side of his pinion. And Marco, living in the red dot town on ***R*** knew nothing about Joseph or ***P***, let alone ***S*** and Richard. Until about 12 ***S***-weeks later, Figure 9, when Joseph met Marco for the first time.

Figure 9 is the universe *relative to Joseph*, where ***P*** is stationary and facing west, and ***S*** and ***R*** rotate around ***P***. Figure 10 is the same universe at the same time relative to Richard, where ***S*** faces east and ***P*** and ***R*** rotate around ***S***. Figure 11 is the universe according to Marco, where ***S*** and ***P*** rotate around ***R***.

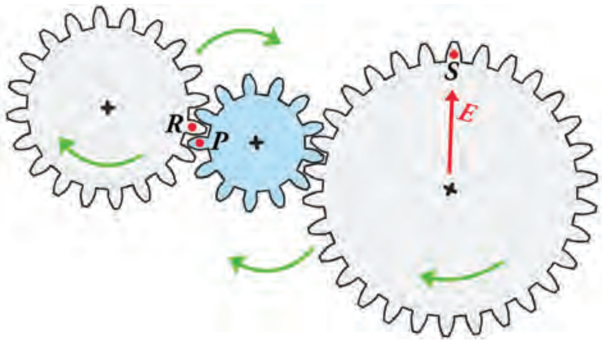

Figure 9

To make it clear that directions, such as east and west, are meaningless, the red arrows on Figures 9, 10 and 11 show the direction of east according to Richard. When God created the universe, Figure 1, he told Richard that east was looking out over the tooth in front of him, and so Richard told Joseph that he faced west. Unfortunately, as can be seen in Figure 9, the relationship between Richard's east and Joseph's west changes continually and, actually, it was impossible for Richard to face east and Joseph to face west simultaneously. In addition, no one told Marco and he arbitrarily chose the direction of the blue arrow to be east.

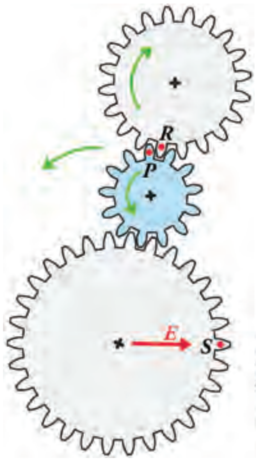

Figure 10

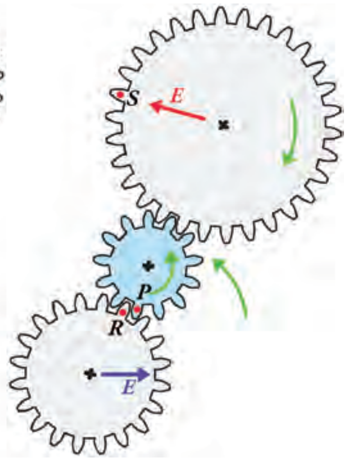

Figure 11

After a few years Joseph and Marco worked out the relationship between ***P*** and ***R***, and a few years later Joseph told Richard about it. Richard was fascinated. From his perspective, Figure 10, ***P*** and ***R*** rotate around ***S*** together. But, because ***P*** is smaller than ***R***, he caught glimpses of something far, far away. And because he and Marco did not like travelling they never met.

God never told anyone that the relationship between the three wheels was fixed by their pivots. And so the doomsday prophets on ***S*** told everyone that ***R*** would rotate around ***P***, collide with ***S*** and destroy the universe. The doomsday prophets on ***P*** then told everyone that ***R*** and ***S*** would be destroyed, but ***P*** would be spared (because God loved ***P*** more than anything else), and this led to a mass migration from ***S*** and ***R*** to ***P***. A conference was held to work out how to feed all the refugees and astronomers were asked to find out if the prophets were right.

As far as everyone in this universe could see, ***S***, ***P*** and ***R*** floated freely in space, because it was impossible to see the pivots from within the universe. And as far as everyone could tell, ***S*** and ***R*** rotated around ***P*** in exactly one ***P***-year. But why was unknown, and any slight discrepancy would cause ***S*** and ***R*** to eventually meet. Also it was not known why ***R*** and ***S*** did not float away from ***P***, presumably because of gravity whatever that is. So a special day of prayer was created. Although Richard, Joseph and Marco were friends, they continually quarrelled about east and who was in the center of the universe. But otherwise everyone lived happily ever after.

2: Mrs God and Planetary Gears

Unfortunately Mrs God was not happy. She wanted to decorate the universe, but other than some chintz curtains there was little opportunity. So God decided to solve the problem and he constructed a new universe, Figure 12, by converting the wheel ***R*** into a ring wheel with its teeth facing inwards. Mrs God was very happy and she draped fairy lights all over ***R*** and called it the firmament. This displeased Marco because he had to change the light bulbs.

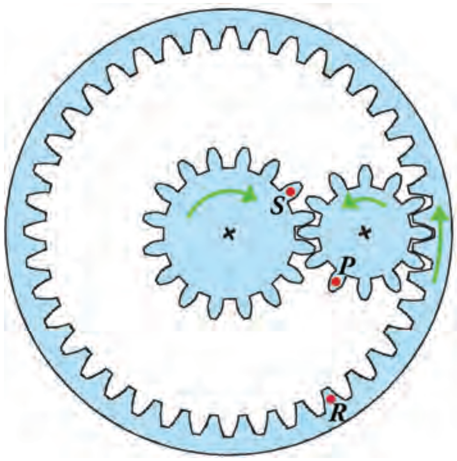

Figure 12

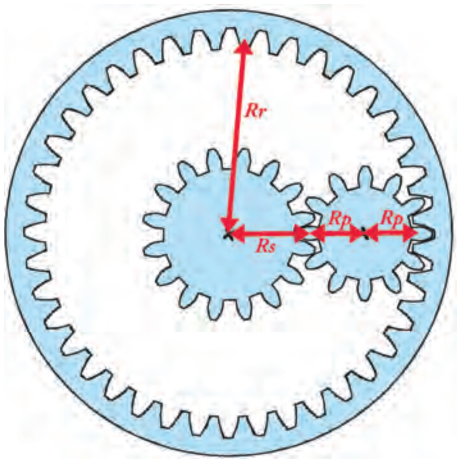

Figure 13

In addition, God changed the teeth counts to something more sensible; ***S***, which he now called the sun, had 16 teeth, ***P***, now called the planet, had 12 teeth, and ***R***, the firmament, had 40 teeth and approximately 228,000,000 light bulbs. This, God realised, was not arbitrary. The teeth on ***S*** had to match the spaces on ***P***, and the teeth on ***P*** had to match the spaces on ***R***; that is, all the teeth and spaces had to be the same size. Also, as shown in Figure 13, the radiuses are related:

$$Rr = Rs + Rp + Rp$$

So God pedantically reasoned thus: If one tooth and one space occupied w and ***S*** had Ns teeth then the circumference of ***S*** must be

$$Cs = wNs$$

and therefore $Cp = wNp$ and $Cr = wNr$

But God knew that $C = 2\pi R$ and so

$$2\pi Rs = wNs,\ 2\pi Rp = wNp \text{ and } 2\pi Rr = wNr$$

So God concluded (correctly) that:

$$wNr/2\pi = wNs/2\pi + 2wNp/2\pi$$

and getting rid of w and 2π resulted in:

$$Nr = Ns + 2Np$$

And God saw that it was good.

He also saw that there must have been some divine intervention, because the universe he created happened to have the right numbers of teeth.

Although God did not realise it at the time, using involute gears was a stroke of luck. Watchmakers knew that with epicycloid teeth "that which drives must be round and that which is driven must be sharp" and hence they usually made the teeth of wheels in the shape of a thumb and the teeth of pinions in the shape of a bay leaf. (Which is why pinions have *leaves*, but wheels should have *thumbs*.) However, this is a problem with the universe: Because ***S*** drives ***P***, ***P*** should have pointed leaves, but ***P*** drives ***R*** and so ***P*** should have round thumbs. However, involute gears do not have this problem.

In this universe ***S*** and ***P*** rotate around pivots in holes in space and ***R*** rotates around a groove in space.

And God saw that it was boring.

Because it was basically the same as the previous design and behaved in much the same way. But ...

First, there was a well defined center of the universe and Marco always faced towards the center. However, directions such as east and west remained meaningless. Figure 12 shows the universe according to God, with east being on the line of centers of ***S*** and ***P***. And Figures 14 to 16 are the same situation, but from the relative point of view of Richard, Joseph and Marco respectively and showing the other three easts.

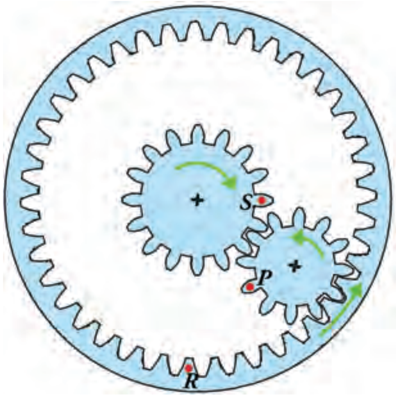

Figure 14

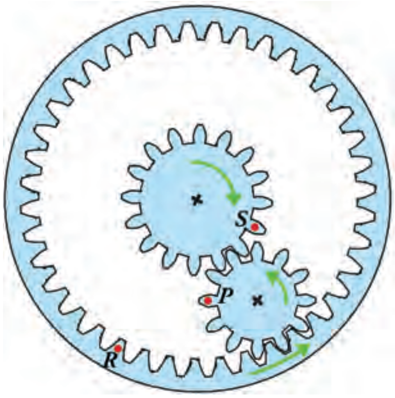

Figure 15

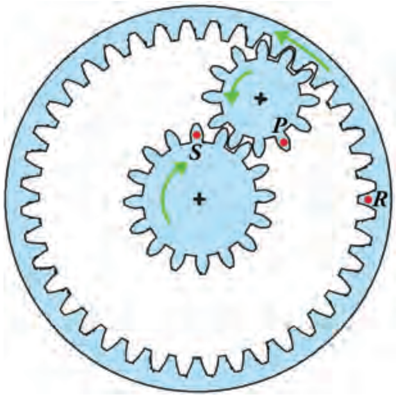

Figure 16

Second, Richard, Joseph and Marco saw each other more frequently.

And third, and much more interesting is that the direction of rotation of ***R*** has reversed; as ***P*** rotates ***R*** rotates with it. So now *Tr = -Ts (Ns/Nr)* whereas it had previously been +*Ts (Ns/Nr)*.

As there was nothing much to do, other than replace light bulbs, while Mrs God was knitting him a Fair-Isle jumper, God decided to look up *planet* on Google, and he found something called *planetary gears*. This looked fascinating, so he immediately changed his universe.

As in Figure 17, he removed ***P***, which terrified Joseph, created a ring ***C*** and put it in a groove in space, made a small hole in ***C***, and then put ***P*** back with its pivot in the new hole.

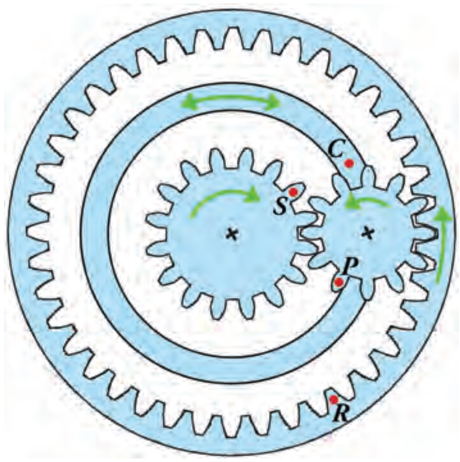

Figure 17

And God saw that it was exciting!

And then God saw that it was unpredictable. God moves in mysterious ways and these gears moved like God, in mysterious ways!

With his previous universes, God imparted motion by using an electric motor he bought from eBay and attached to ***S***. Sometimes put his finger on ***P*** or ***R*** and stopped it rotating. And the result was that everything stopped.

But this new universe was different. With the motor on ***S***, when God put his finger on ***C*** he found that ***S***, ***P*** and ***R*** continued to rotate, and when he put his finger on ***R*** he found that ***S***, ***C*** and ***P*** continued to rotate. But if he stopped ***P*** from rotating everything locked and ***S***, ***C*** and ***R*** rotated together. Strange. Even stranger, if he moved ***C*** clockwise or anti-clockwise ***P*** and ***R*** rotated in different ways. So God added an electric motor to ***C*** with a controller from a model train set so that its rotation could be adjusted independently.

The first important discovery was that when he put his finger on ***C***, so that $Tc = 0$, then ***S***, ***P*** and ***R*** behaved exactly as they had done before. This time God wrote the formulae as:

$Tr = Tp\ (Np/Nr)$ and so $NrTr = NpTp$

$Tp = -\ Ts\ (Ns/Np)$ and so $NsTs = -\ NpTp$

Therefore:

$$NrTr + NsTs = NpTp - NpTp = 0$$

But what happened when ***C*** moved? Simon, who you have not met before but who lives on ***C***, was no help at all. According to him ***C*** was stationary, but ***P*** sometimes rotated clockwise and sometimes anti-clockwise, and he had no idea why. However, Joseph thought ***P*** was stationary and ***C*** pirouetted around him, as in Figures 18 to 21. God decided the universe surpasseth all understanding, got a migraine and went to bed.

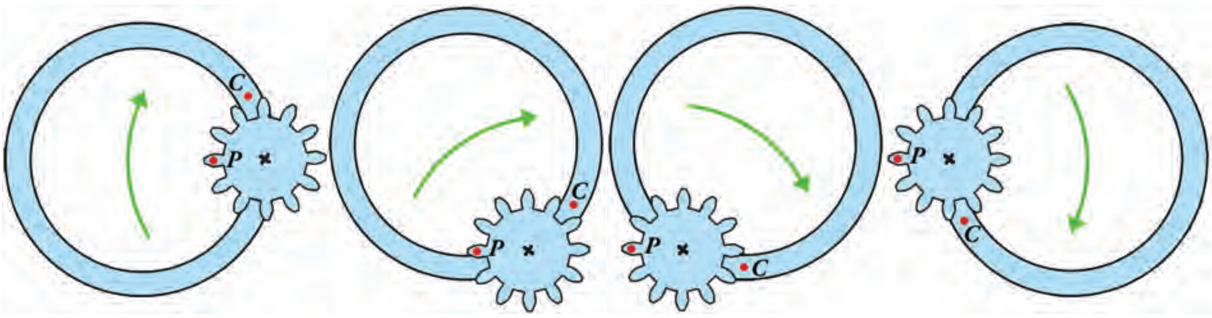

Figure 18 Figure 19 Figure 20 Figure 21

Next morning, God realised that Richard, Joseph, Marco and Simon were not going to help, and it was up to him to work out what was happening. When God had created the previous formulae, he had noted that ***P*** vanished from them and Joseph was visibly upset, because it seemed no one loved him. So God thought he would work out what was happening to ***P*** and try to make ***P*** vanish again. To do this he decided to look at the universe keeping ***P*** to the east of ***S***, as in Figure 17, and see how ***S***, ***P***, ***C*** and ***R*** rotated.

First God put a motor on ***S*** and considered how ***P*** rotated relative to it. Obviously ***P*** rotated because of the two motors on ***S*** and ***C*** and so Tp must be the sum of these influences. Considering the rotation of ***S*** by itself:

$Tp = -\ (Ns/Np)\ Ts$

because ***S*** is rotating ***P*** in the opposite direction.

Also, ignoring the motion of ***S***, ***P*** has to rotate in the same direction as ***C*** rotates around ***S***, because of its teeth meshing with ***S*** (which it can do by turning ***R***). As a single turn of ***C*** caused ***P*** to pirouette once around ***S*** and so:

$Tp = (Ns/Np)\ Tc$

Therefore, the *actual* rotation of ***P*** must be the sum of these two influences and:

$Tp = (Ns/Np)\ Tc - (Ns/Np)\ Ts$

Because God wanted to get rid of ***P***, he rewrote this as:

$NpTp = NsTc - NsTs$

He also noted that ***R*** was irrelevant because it followed the other movements.

At this point God got stuck; he simply couldn't work out how to get ***R*** into the formula. But then Marco complained, because *he* was in the center of the universe, so why should he miss out. God could see his point and realised that either ***S*** or ***R*** could be the "center" and so he added an electric motor to ***R***. After a few motors had exploded, God realised that either ***S*** or ***R*** could be driven but not both.

So he put the motor on ***R*** to see what ***P*** did relative to ***R***. He immediately realised that the behaviour was the same, except for different numbers of teeth, and now ***S*** was irrelevant.

Considering the rotation of ***R***:

$Tp = (Nr/Np)\ Tr$ or $NpTp = NrTr$

because the internal teeth of ***R*** rotate ***P*** in the same direction.

Also, ***P*** has to rotate in the opposite direction to ***C*** as ***C*** rotates around ***R***, because of its teeth meshing with ***R***. A single turn of ***C*** caused ***P*** to pirouette once around ***R*** and so:

$Tp = -\ (Nr/Np)\ Tc$ or $NpTp = -NrTc$

Again adding the two together, the *actual* rotation ***P*** is:

$NpTp = NrTr - NrTc$

Now, at last, God could get rid of ***P*** and Joseph:

$NsTc - NsTs = NrTr - NrTc$

In other words:

$NsTc + NrTc = NrTr + NsTs$

As God wrote this down there was crash of thunder and a tablet materialised reading "Algebra: 95%, High distinction". And Joseph was mollified, because this was the first time anyone had explained how ***P*** moved.

Some time later God read another explanation of planetary gears: Consider the motion of ***S*** and ***R*** *relative* to ***C***; that is, find the ratio of the turns of ***S*** and ***R*** with respect to ***C***: $(Tr - Tc) / (Ts - Tc)$. this ratio is determined by the number of teeth of the wheels and is:

$- NsNp / NpNr = - Ns / Nr$

This is is negative because ***S*** and ***R*** rotate in opposite directions. That is:

$(Tr - Tc) / (Ts - Tc) = - Ns / Nr$ which is the same as above.

Although it is a much simpler explanation, God preferred his approach.

3: Breguet and Trigonometry

Most 18th and early 19th century self-winding watches use stop-work mounted on the mainspring barrel to lock the weight when the mainspring is fully wound. This stop-work, that acts on a lever, or a system of levers, is quite often the normal Geneva or Maltese-cross stop-work with a minor modification. They consist of a wheel mounted on the barrel arbor with a single tooth, commonly called the *finger*, and a wheel mounted on the barrel with a number of slots. Explanations of these can be found in *"Origins"* (Watkins, 2016, Appendix 7) and animations of them are available from http://www.watkinsr.id.au/Animations/Animations.html.

Although precise information on Breguet's stop-work has never been published, it appears that he used at least two designs created by himself. One design is known from photographs of Breguet's watch No. 28 in the collection of the Musée international d'horlogerie, La Chaux-de-Fonds, Switzerland. There are apparently *three* watches with the number 28; the MIH watch discussed here, circa 1791, Sabrier (2012, page 91), circa 1784, and Daniels (1975, page 146), circa 1791. All have obvious differences. The other, apparently later design is described by Breguet in an undated manuscript that was probably written in the early 19th century. Translations of the manuscript appear in Chapuis & Jaquet (1956, pages 107-108) and Sabrier (2012, pages 84-85). These designs also consist of a wheel mounted on the barrel arbor with a finger, and a wheel mounted on the barrel with a number of slots.

For reasons that will become apparent, I will examine Breguet's designs in reverse order, starting with his manuscript. Instead of quoting his complete words, which are quite precise, I will explain the mechanism using Figure 22; it shows the position when the mainspring is fully wound.

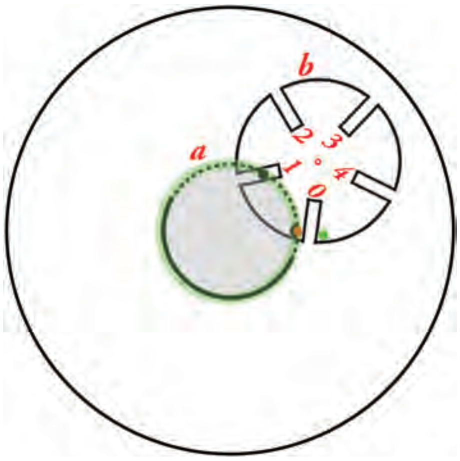

Figure 22

(a) The barrel wheel ***b*** is a crown wheel with five equidistant slots. The diameters of the arbor wheel ***a*** and the barrel wheel ***b*** are in the ratio 4:5.

(b) The barrel wheel has two pins on either side of slot 0. The red pin is the *boss* that stops *winding*; it extends above the arbor wheel to raise the green annulus to lock the weight. The green pin stops *unwinding* and its top is level with the outside of the arbor wheel. It prevents the red boss reaching the annulus when the mainspring is unwound,

Breguet is a bit vague regarding the positions of these two pins, writing *this barrel wheel carries a strong steel pin inserted in the bottom and quite near to its crown* (the red boss pin) and *there is another, lower pin, placed a little in front of the long one and which alone stops the bottom of the spring*. Placing them on either side of the slot fits this description.

(c) The annulus, the transparent green circle, is slightly larger than the arbor wheel. It and the arbor wheel are transparent so that the barrel wheel and its two pins underneath are visible.

(d) The arbor wheel is also a crown wheel. The base of the arbor wheel is not cut and forms a complete circle. About half of its crown is removed but leaving a small piece for the finger that reaches down into the slots of the barrel wheel. The solid part of the circumference of ***a*** is the section that has the crown. And, except for the finger, the dotted part of the circumference of ***a*** is the section that has the crown removed.

Breguet actually specifies this design, writing *one part of the crown* [not the surface] *of the smaller, upper wheel is cut away*, and *this pin* [the boss] *butts against the circumference of the small arbor wheel.*

(e) Breguet notes that when the red pin butts against the arbor wheel *it even half penetrates it through a slot made for this purpose*; Figure 22 shows this slot and the boss in it.

This feature is necessary. Without it the boss can only reach the outermost part of the annulus and it is possible that the weight will not be locked. With it, more of the annulus is accessible.

(f) In *"Origins"* (page 120) I note that the two barrels used by Breguet rotate anti-clockwise to run the watch and, so that the hands turn in the correct direction, there is an intermediate wheel and pinion between the barrels and the center pinion.

(g) Breguet states that the five slots allow the arbor wheel to make four revolutions while the barrel wheel makes only ⅗th of a revolution. He is correct regarding the latter, but wrong regarding the former.

The finger rotates the barrel wheel ⅕th turn every time it enters a slot; that is, for every turn of the arbor wheel. The fully wound position in Figure 22 has the finger in slot 1, and successive turns of the mainspring place the finger in slots 2, 3 and 4; the finger can never enter slot 0. That is, this design allows only three (and a bit) turns of the mainspring.

Figures 23 to 25 show the positions after the mainspring has unwound 1, 2, and 3 turns. At this point the mainspring can unwind about a further ⅕ turn before the green pin butts against the arbor wheel. To achieve four turns requires a different design, or a barrel wheel with 6 slots.

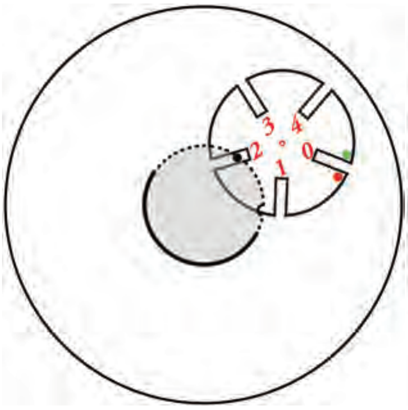

Figure 23

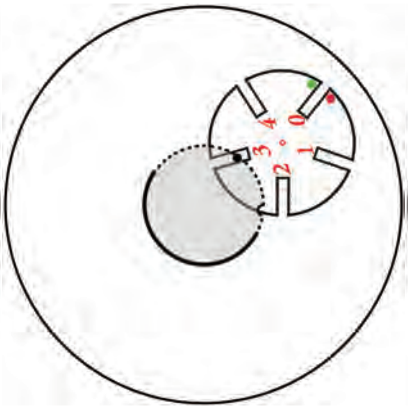

Figure 24

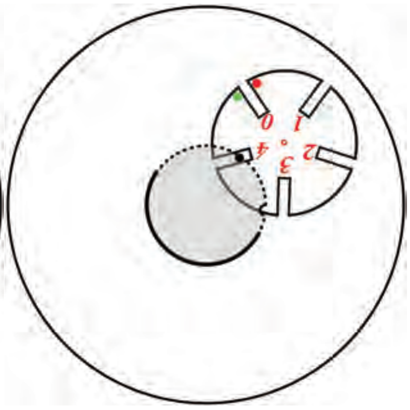

Figure 25

These pictures are strange. We know that the mainspring unwinds when the barrel rotates anti-clockwise, but they show the barrel stationary with only the arbor and barrel wheels rotating. This is because they are *views relative to the barrel*.

At this point it is again useful to re-introduce relativity, because the views of God, Richard (sitting on the arbor wheel), Joseph (sitting on the barrel) and Marco (sitting on the barrel wheel) are quite different. There are six different views in two groups:

(a) http://www.watkinsr.id.au/Animations/Animations-Relativity/Animations-Relativity-Winding.html: This shows the three views when the arbor wheel rotates anti-clockwise to wind the mainspring. The *normal*, most useful view is that of God, which is also the view seen by Joseph sitting on the barrel, whereas the views of Richard and Marco are dramatically different.

The last animation is the most interesting. God felt guilty because Marco had to spend an eternity changing light bulbs. So for his next eternity God provided him with an arm chair, unlimited cigars and single malt whiskey, and the best view in the universe. The animation is also interesting because it shows the dramatic change produced by a different, relative point of view.

(b) http://www.watkinsr.id.au/Animations/Animations-Relativity/Animations-Relativity-Unwinding.html: This shows the three corresponding views when the barrel rotates anti-clockwise to unwind the mainspring and run the watch. The most useful view is also that relative to the barrel when the arbor wheel relatively rotates clockwise.

That is how Breguet's 2-pin design works. But is it correct?

Figure 26 presents the mathematical ideal. The barrel wheel, with a radius ***Rb*** of 2.5 mm, is divided into 5 *sections* of 72°; a *segment* is the angular width of the barrel wheel crown between slots, and a *section* is the angular width of a segment and an adjacent slot. For convenience, I have made the dimensions a little larger than the designs that follow. The arbor wheel is drawn and positioned so that its crown enters and leaves the sections perpendicular to the tangents at those points. From the right-angle triangle formed by the two radiuses and the line connecting the two centers:

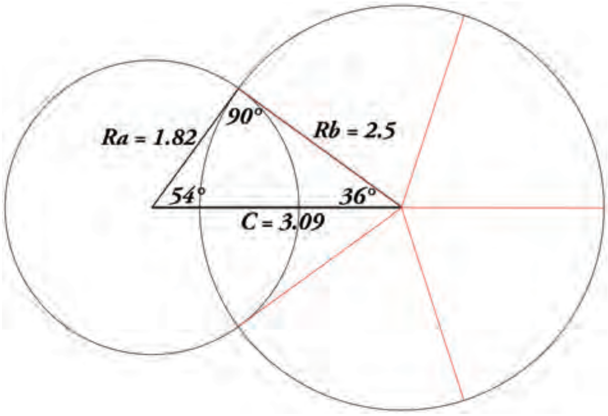

Figure 26

$$Ra = Rb \tan(36°) = 1.816$$

$$C = Rb/\cos(36°) = 3.090$$

By using an arbor wheel with a radius of ***Ra*** = 2 mm, Breguet has introduced an error of about 10%, probably not significant, and it is clear that an arbor wheel with a radius of 2 mm will work successfully with a barrel wheel with a radius of 2.5 mm.

Trigonometry is not needed to design these wheels, and the following uses only a ruler and compass, noting that the 5 slots in the barrel wheel form a pentagon.

A simple method for inscribing a regular pentagon inside a circle is described by Martin (1759, page 330); see Figure 27:

(a) Black: The diameter of the circle ***C1*** is drawn, the perpendicular at ***A*** is found and the radius ***OA*** is drawn.

(b) Green: The midpoint ***B*** of ***OC*** is found. Using ***B*** as the center, the arc ***AL*** is drawn.

(c) Red: Set the compass to the distance between ***A*** and ***L*** and, using ***1***, ***2***, ***3*** and ***4*** as centers, draw arcs at ***2***, ***3***, ***4*** and ***5***.

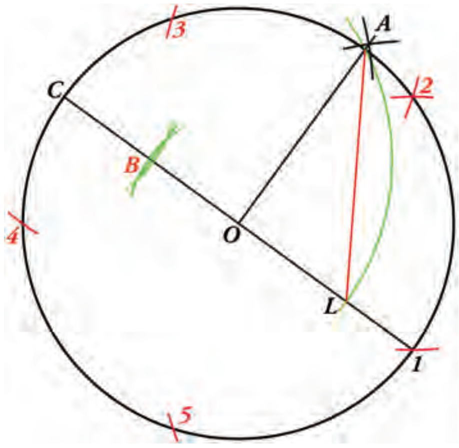

Figure 27

Proving that this is correct is also simple, with a little help from Pythagoras; see Figure 28:

R is the radius of the black circle and ***R'*** is the radius of the green circle. The line ***15*** of length ***P*** is a side of the pentagon.

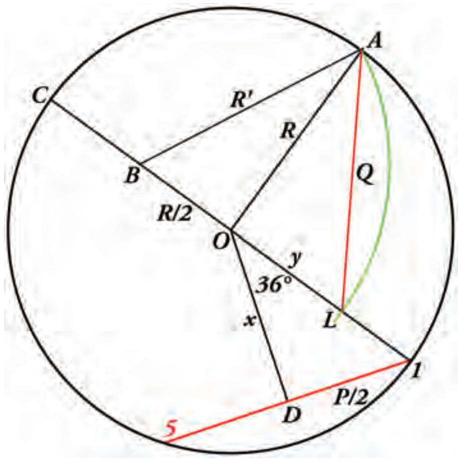

Figure 28

First, consider the value of ***P***. From the triangle ***OD1***:

$$R^2 = x^2 + (P/2)^2$$

Also, Wikipedia shows that $\cos(36°) = (1+\sqrt{5})/4 = x/R$ and so

$$x = R(1+\sqrt{5})/4$$

Substituting and simplifying:

$$R^2 = (R(1+\sqrt{5})/4)^2 + (P/2)^2$$

$$P^2/4 = R^2 - (R(1+\sqrt{5})/4)^2$$

$$P^2 = 4R^2 - 4(R(1+\sqrt{5})/4)^2$$

$$= 4R^2 - 4R^2(1 + 2\sqrt{5} + 5)/16$$

$$= R^2(5/2 - \sqrt{5}/2)$$

Second, the value of ***Q***, the length of the line ***AL***. From the triangle ***OAL***:

$$Q^2 = R^2 + y^2$$

Now $BL = R'$ and so $y = R' - R/2$. From the triangle *OAB*:

$$R'^2 = R^2 + (R/2)^2 \text{ and } R' = \sqrt{(R^2(1 + ¼))} = (R\sqrt{5})/2$$

$$y = (R\sqrt{5})/2 - R/2 = R/2(\sqrt{5} - 1)$$

So:

$$Q^2 = R^2 + (R/2(\sqrt{5} - 1))^2$$

$$= R^2 + R^2/4(5 - 2\sqrt{5} + 1)$$

$$= R^2 + R^2 3/2 - R^2\sqrt{5}/2$$

$$= R^2(5/2 - \sqrt{5}/2) = P^2$$

Because I am not a mathematician and God did not help me, this took me about as many days as a mathematician would need minutes to solve.

Rather than using trigonometry, the center of the arbor wheel can easily be found using geometry; Figure 29:

(a) Red: Using ***1*** and ***2*** as centers, draw the compass marks ***A*** and draw ***OC*** through A.

(b) Green: Draw the line ***OB*** through ***2***. With the center at ***2*** draw two arcs on ***OB***. Using these intersections as centers, draw the compass marks ***L*** and draw the line ***2D*** through ***L***; this is the tangent line to the barrel wheel.

(c) Yellow: Using the intersection of ***OC*** and ***2D*** as the center and ***2D*** as the radius, draw the arbor wheel.

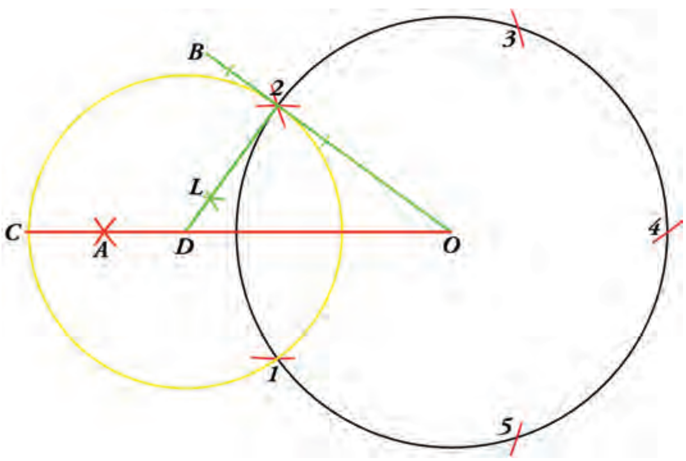

Figure 29

Measuring this diagram with a ruler and scaling the numbers to a barrel wheel radius of 5 gives an arbor wheel radius of 3.7 and a center distance of 6.2, very close to the correct values.

Breguet's earlier design was used in his watch No. 28, and Daniels (1975, pages 344-345) appears to describe the same design. The main feature is that the barrel and arbor wheels have the same diameter of about 4.24 mm. Also, this design uses a boss, a raised section on a segment of the barrel wheel, instead of two pins to control winding and unwinding, and so it is similar to Geneva and Maltese-cross stop-work.

Figure 30 is the abstract model where the center to center distance ***C*** is:

$$\boldsymbol{C} = 2\boldsymbol{Rb}\,cos(\boldsymbol{\mu}) \text{ and}$$

$$\boldsymbol{\mu} = acos(\boldsymbol{C}/2\boldsymbol{Rb})$$

The latter is important because the number of slots is

$$180/\boldsymbol{\mu}$$

For 5 equally spaced slots $\boldsymbol{\mu}$ = 36° and, with ***Rb*** = 2.12, ***C*** = 3.44 mm.

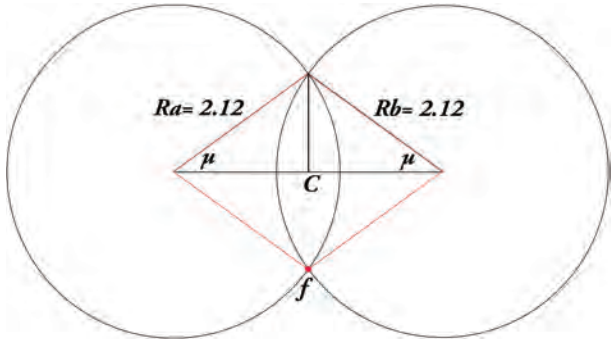

Figure 30

However, Breguet's watch No 28 is completely different. Figure 31 is a photograph of the wheels in this watch, showing that the slots in the barrel wheel are relatively very wide. (Note that in this photograph the arbor wheel is in an "impossible" position where the finger ***f*** is resting on the base of the barrel wheel instead of being in a slot. So it is likely that the finger has been damaged.) Most important is that the center distance between the wheels is much smaller than the distance predicted by Figure 30.

Figure 31

Figure 32

When I wrote *"Origins"*, my analysis of watch No 28 was based on animations, and at that time I did not realise that the animation showing the mainspring being wound was wrong. Figure 32 shows the position part way through winding in the original animation, and it is clear that the arbor wheel is blocked by the crown on the barrel wheel and it cannot rotate anti-clockwise. The mainspring can only be wound about one turn. This created much anxiety, because it seemed possible that my entire explanation of the watch might be wrong.

At this point I asked God to help, but he refused. "I got a migraine last time I helped and I am not having that again. Anyway, I have to work with Marco, because Orion has gone out." (When this was fixed, astronomers were upset because Betelgeuse had changed colour from red to green.) However, God, concerned by my psychological state, suggested that I should not let the truth stand in the way of a good story. After all, no one will notice that *"Origins"* is wrong and it is likely that no one would read this article. So let sleeping dogs lie. However, despite this reassurance I felt uncomfortable, so I spent several very frustrating days trying to understand the stop-work. I tried several animations with various center distances ***C***, but all failed catastrophically!

Eventually I found that the only animation that worked had a center distance of about ***C*** = 3.00 mm instead of 3.44 mm, which is much too small! (http://www.watkinsr.id.au/Animations/Animations-Breguet-No28/Animations-Breguet-No28.html)

In Figure 33 the blue circle is the arbor wheel corresponding to this value of ***C*** and the green ***f*** is the finger. The green lines ***1*** and ***2*** are two of the five slots in the barrel wheel separated by 72°. When the finger rotates the barrel wheel these slots move to the positions of the red lines, after which the arbor wheel is free to rotate until its finger again meets the barrel wheel. But from Figure 30, μ = *acos(3.00/4.24) = 44.96*°, so the slots rotate about 90°, 18° too far, and the finger butts against the barrel wheel instead of entering slot ***2***.

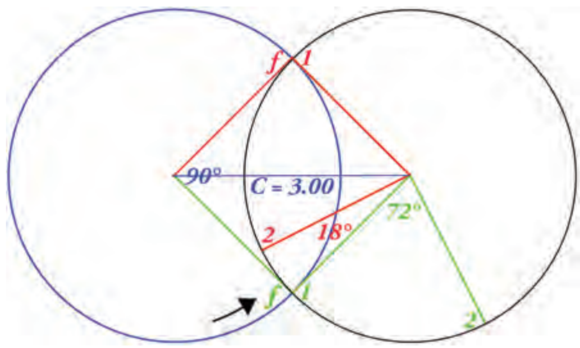

Figure 33

It is clear that this design cannot work. But watch No 28 does work!

The solution to this enigma lies in the difference between the mathematical ideal and the practical reality. Figure 33 represents the wheels with infinitely thin slots and an infinitely small finger. What difference does it make when the *real* wheels in Figure 31 are used?

Figure 34 gives the angular dimensions of the barrel wheel that is not equally divided. (Parts of two segments and one slot were hidden beneath the arbor wheel, but enough is visible to estimate the shapes of these.) Four sections (segments and their slots) cover 70°, 50° for the segments and 20° for the slots. The fifth segment and slot cover 80°.

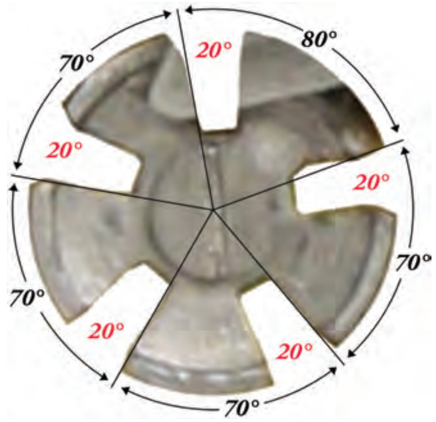

Figure 34

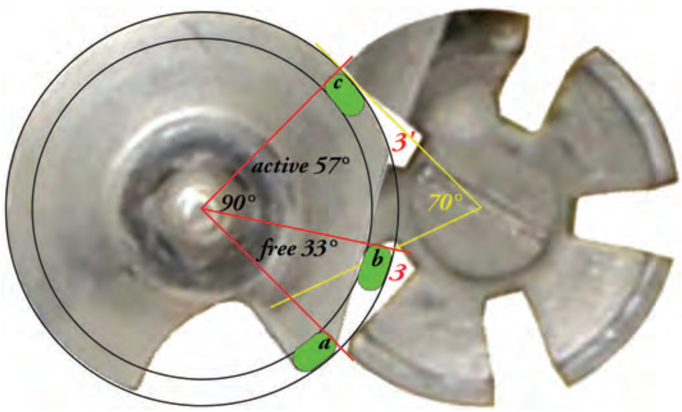

Figure 35

Figure 35 shows what happens when the mainspring is being wound and the arbor wheel is rotating anti-clockwise. Initially the finger is at ***a***, and the arbor wheel rotates until the finger reaches ***b*** *without moving the barrel wheel*. Then the arbor wheel rotates the finger to the position at ***c*** turning the barrel wheel clockwise so that slot 3 moves from bottom left to 3' top left. From that point the arbor wheel rotates until the finger returns to the position ***a***. It can do this because the crown, indicated by the two black circles, moves through the slots.

During this process, the arbor wheel rotates through 90°, indicated by the red lines at ***a*** and ***c***, but the barrel wheel only rotates through 70°, indicated by the two yellow lines. The difference is due to the free movement of the finger between ***a*** and ***b***. (The angles are 33° and 57° because how far the finger rotates the barrel wheel depends on the depth of penetration of the finger in the slot.)

The width of the slots is very important. To make this clear, in the following figures the arbor wheel is transparent to show the barrel wheel slot underneath it.

Figures 36 and 37 show the behaviour when the slots are narrower, about 15° instead of 20°.

Figure 36 shows one position. Here the finger cannot enter slot 3 because its rotation is blocked by the barrel wheel crown. And the barrel wheel cannot rotate anti-clockwise so that the finger can enter the slot because the arbor wheel crown in slot 4 prevents this rotation.

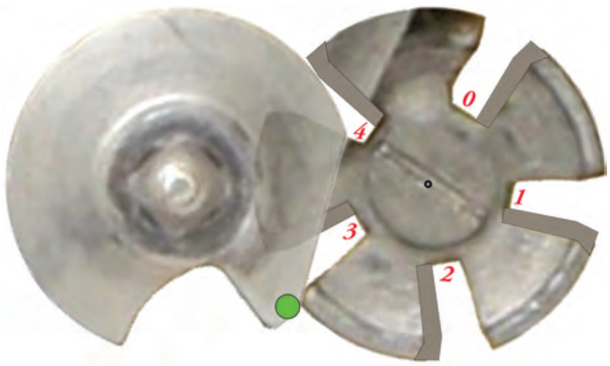

Figure 36

Figure 37 shows another impossible position. Here the finger in slot 3 is trying to rotate the barrel wheel clockwise, but the arbor wheel crown in slot 2 prevents this rotation.

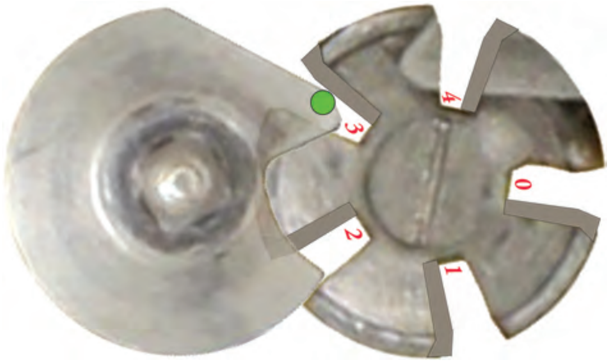

Figure 37

15° slots will work, as in Figure 38, by moving the barrel wheel away from the arbor to increase the center distance. The barrel wheel appears to have moved too far, but the open mouths of the slots are misleading, as shown by the dotted lines.

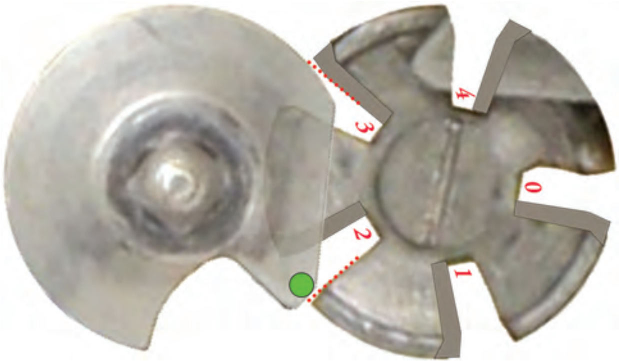

Figure 38

Figure 39 shows what is happening. The initial position is with the red finger $\boldsymbol{f}$ leaving the red *leading edge* of slot 1. The arbor wheel then rotates anti-clockwise until the finger (green $\boldsymbol{f}$) reaches the green *trailing edge* of slot 2. In order for the finger to enter slot 2 the angle between its leading and trailing edges must be at least 20° (green shading). The red shading shows the same width between the leading and trailing edges of slot 1.

In the watch, Figure 34, the angle between the leading edge of slot 1 and trailing edge of slot 2 is 90°. In Figure 30 $\boldsymbol{\mu} = 45°$, and the calculated center distance is $\boldsymbol{C} = 2\boldsymbol{Rb}cos(45°) = 3.00$, the same as my experimental center distance. Also, the 90° angle means that the finger and the rim enter the slots perpendicular to the tangents, the theoretically ideal arrangement.

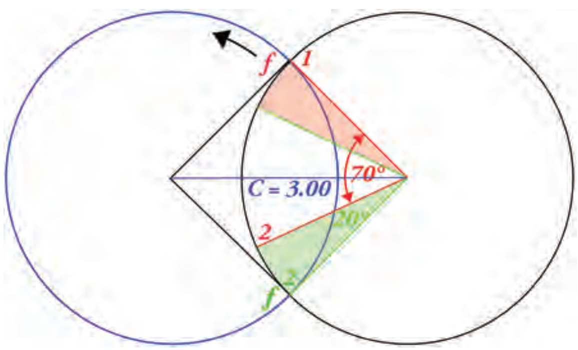

Figure 39

However, reducing the slots to 15° changes the angle between the leading edge of slot 1 and trailing edge of slot 2 to 85°, and so in Figure 30 $\boldsymbol{\mu} = 42.5°$ and $\boldsymbol{C}$ increases to 3.13, which is the center distance I used in Figure 38.

In theory the slots can be any width greater than 20° and Figure 40 shows slots of 40°.

In this case the finger will rotate the barrel wheel correctly, because it *must* rotate the leading edge of slot 2 to the position of the leading edge of slot 1 in the diagram.

This is illustrated by the Breguet 2-pin animations used to demonstrate relativity, where the slots are wider than the slots in the other animations at:

http://www.watkinsr.id.au/Animations/Animations-Breguet-2-Pin/Animations-Breguet-2-Pin.html.

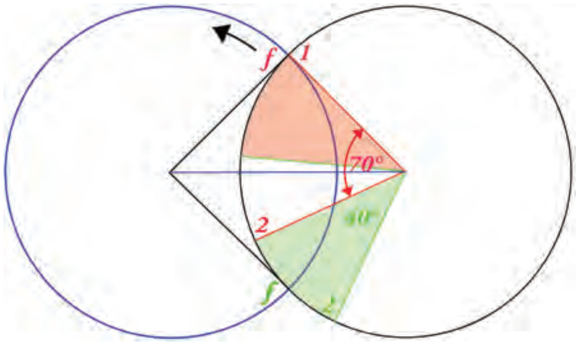

Figure 40

However, such wide slots will weaken the barrel wheel making damage to it much more likely. Also, the barrel wheel now has considerable freedom, being able to rotate about 20° clockwise.

Figure 41 shows the stop-work wheels of another Breguet self-winding watch, circa 1810. There has been catastrophic damage, including breaking the finger and the barrel wheel. Note that the arbor wheel has the same design as that of watch No. 28, and the barrel wheel is better made; from the regularity of the slots, it was probably made on a wheel cutting engine.

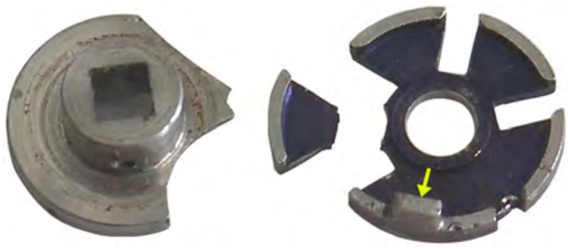
Figure 41

Most important is that the position of the boss on the barrel wheel is completely different, being in the middle of the segment and not at one end. And one slot has not been made and only the rim has been removed.

The division of the barrel wheel, Figure 42, is very different from that in watch No. 28, shown in Figure 34:

(a) Three slots are 15° wide and two are 16° wide, instead of 20°.

(b) Three segments are 67° wide and one is 66° wide instead of all segments being 70° wide.

(c) The segment with the boss is now 93° wide instead of 80°.

The small variations involve the broken segment and may be caused by errors in the photograph.

Figure 42

Animations of this design are available from http://www.watkinsr.id.au/Animations/Animations-Breguet-Other/Animations-Breguet-Other.html. Although I have some measurements of this watch I do not know the diameter of the annulus.

Compared with watch No. 28, there is one very important difference, the number of turns made by the mainspring.

The animations of No.28 show that the mainspring can be wound 3.07 turns. However, if the annulus is smaller, then the mainspring can be wound 3.88 turns.

In contrast, the other watch winds 3.88 turns and, if the annulus is smaller, then the mainspring can be wound 3.92 turns.

This difference is due to one feature, *the position of the boss*. In the other watch, by placing the boss in the middle of the segment (instead of at the end) ensures about 4 turns.

The second important feature of the other watch is the uncut slot on the barrel wheel. As the animations show, the crown at this slot must be removed, because the crown of the arbor wheel must enter it. But what happens if the slot is cut out?

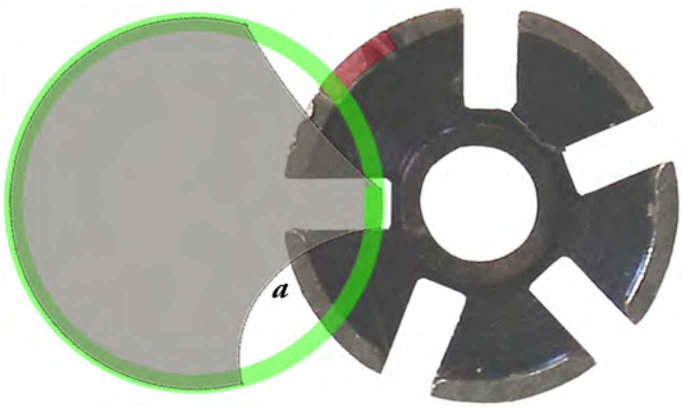

Figure 43

Figure 43 shows what happens with a large annulus. The barrel will continue rotating until the boss lifts the annulus and locks the weight. (Actually, the mainspring will unwind further until the boss butts against the arbor wheel.)

This is catastrophic. The watch will not run, because the mainspring no longer supplies power to the train, and the mainspring cannot be wound, because the weight is locked. So the slot cannot be cut out.

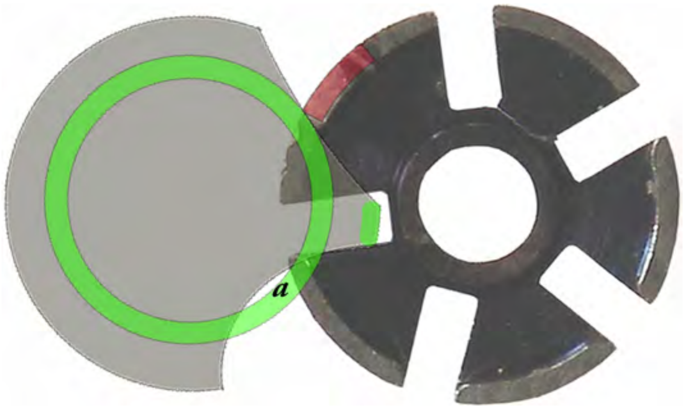

Figure 44

Figure 44 shows what happens with a small annulus. The barrel will continue rotating until the boss butts against the arbor wheel, but it will not raise the annulus. The weight remains unlocked and the mainspring can be wound.

As noted above, cutting the slot to its full depth only produces a very small increase in the number of turns of the mainspring and it is not necessary.

Also note that the cut-out part of the arbor wheel at ***a*** is necessary, because the crown has to be removed from this part of the wheel. But with a large annulus the boss never enters this region and the base of the arbor wheel could be uncut, as in the 2-pin design. In contrast, with a small annulus this part must be removed to allow the boss to reach the annulus. As this other watch probably has a large annulus, like watch No. 28, the shape of the cut-out ***a*** is not important.

Figure 45 is the diagram of this other watch and is the equivalent of Figure 39 for watch No. 28. It is important to understand how this figure was created from the measurements provided by Anthony Randall:

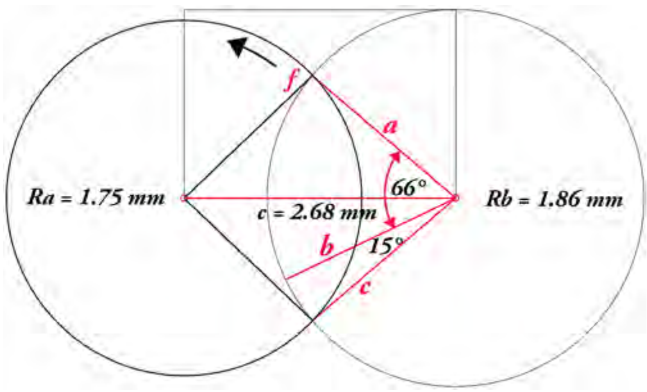

Figure 45

(a) A rectangle was created as wide as the center distance between the wheels and as high as the barrel wheel radius.

(b) Two circles of the diameters of the arbor and barrel wheels were made with their center positions defined by the rectangle.

(c) The line ***a*** was drawn to the intersection of the two circles. This is the leading edge of a slot and the finger ***f*** is about to leave it.

(d) Line ***a*** was duplicated, rotated through 66° and positioned at ***b***. This is the leading edge of the next slot, based on Figure 42.

(e) Line ***b*** was duplicated, rotated and positioned at ***c***. The rotation was not copied from Figure 42, it was done by trial and error and the angle was found to be 15°. This is the trailing edge of the slot. At this point the barrel wheel is completely defined and all the slots can be added, creating Figure 42.

As with watch No. 28, this diagram confirms that, for the dimensions of this other watch, the design is correct.

4: Breguet and Equality

And God, who surpasseth all understanding, spake: "Liberté, égalité et fraternité."

"What in Heaven do you mean! ! ! ? ? ?"

"Well, Breguet was born in Switzerland, which is as good as being French, and lived for most of his life in Paris, so the French motto is appropriate."

"I have no idea what you are talking about."

"Equality. Why don't Breguet's watches have equal segments?"

"OK, I admit that is an important question. So ..."

It is easy to show that Breguet's watch No. 28 will not work if all segments are equal as shown in:

http://www.watkinsr.id.au/Animations/Animations-Equal-Segments/Animations-Breguet-No28-Equal-Segments.html

Starting with the mainspring fully unwound, the animation shows that after winding for about one turn the crown of the arbor wheel butts against the crown of the barrel wheel. At this point the arbor wheel cannot rotate but, because the weight is unlocked, the self-winding mechanism will continue to operate, which is catastrophic.

However, it is easy to modify the arbor wheel so that winding can continue, by cutting back the crown.

The unwinding animation is interesting because, with or without cutting back the crown, the mainspring will unwind correctly. This is because the arbor wheel is asymmetric.

However, it is also easy to show that Breguet's other watch does function correctly with equal segments:

http://www.watkinsr.id.au/Animations/Animations-Equal-Segments/Animations-Breguet-Other-Equal-Segments.html

Why?

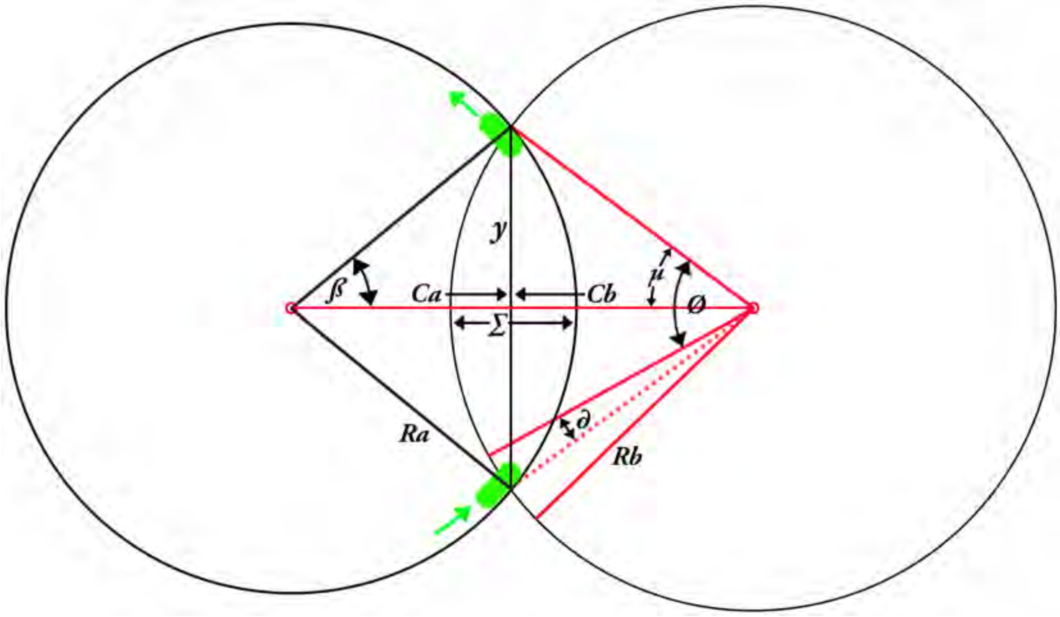

Figure 46

Figure 46 (not to scale) can be used to calculate the dimensions of any stop-work. In it, the arbor wheel is rotating anti-clockwise to wind the mainspring. The finger (green) leaves the top slot and then enters the bottom slot as the arbor wheel rotates anti-clockwise.

The angular dimension of a section is $\boldsymbol{\emptyset}^\circ$; that is, $\boldsymbol{\emptyset}$ is the width of a segment and a slot.

The width of a slot is $\boldsymbol{w}$ and so the width of a segment is $\boldsymbol{\emptyset} - \boldsymbol{w}$.

The intersections of the arbor and barrel wheels encompass $\boldsymbol{\emptyset}^\circ + \boldsymbol{\partial}^\circ$, where $\boldsymbol{\partial}$ (dotted line) is the angular width between the leading edge of a slot and the trailing edge of the finger. The width of the slot $\boldsymbol{w}$ (solid lines) must be equal to or greater than $\boldsymbol{\partial}$. The important value $\boldsymbol{\mu} = (\boldsymbol{\emptyset} + \boldsymbol{\partial})/2$.

From the dimensions of the barrel wheel:

$\boldsymbol{y} = \boldsymbol{Rb}\sin(\boldsymbol{\mu})$ (where $\boldsymbol{Rb}$ is the radius of the barrel wheel)

$\boldsymbol{Cb} = \boldsymbol{Rb}\cos(\boldsymbol{\mu})$ (where $\boldsymbol{Cb}$ is the distance from the vertical $\boldsymbol{y}$ to the center of the barrel wheel).

Then, from the dimensions of the arbor wheel:

$\sin(\boldsymbol{\beta}) = \boldsymbol{y}/\boldsymbol{Ra}$, or $\boldsymbol{\beta} = \mathrm{asin}(\boldsymbol{y}/\boldsymbol{Ra})$

$\boldsymbol{Ca} = \boldsymbol{Ra}\cos(\boldsymbol{\beta})$

Thus $\boldsymbol{C} = \boldsymbol{Rb}\cos(\boldsymbol{\mu}) + \boldsymbol{Ra}\cos(\boldsymbol{\beta})$

These trigonometric formulae require us to know $\boldsymbol{Ra}$, $\boldsymbol{Rb}$, $\boldsymbol{\emptyset}$ and $\boldsymbol{\partial}$ (and hence $\boldsymbol{\mu}$) to calculate $\boldsymbol{Ca}$ and $\boldsymbol{Cb}$ and consequently $\boldsymbol{C}$.

Bolt of lightning, clap of thunder, stone tablet “Pythagoras!”

“Why?”

“I got bored waiting for you to see it.”

When we know $\boldsymbol{C}$ Pythagoras can be used directly to calculate $\boldsymbol{Ca}$ and $\boldsymbol{Cb}$ and hence $\boldsymbol{\mu}$. And if we know $\boldsymbol{\emptyset}$, then we can calculate $\boldsymbol{\partial}$:

$\boldsymbol{y}^2 = \boldsymbol{Ra}^2 - \boldsymbol{Ca}^2 = \boldsymbol{Rb}^2 - \boldsymbol{Cb}^2$

That is:

$$Ra^2 - Ca^2 = Rb^2 - (C - Ca)^2 = Rb^2 - C^2 + 2C\,Ca - Ca^2$$

and consequently:

$$Ca = (Ra^2 - Rb^2 + C^2)/2C$$

Also $Cb = (Rb^2 - Ra^2 + C^2)/2C$

As we know Cb we know $\mu = \mathrm{acos}(Cb/Rb)$.

In addition, Σ is the minimum depth of the slots, the distance that circumference of the arbor wheel penetrates into the barrel wheel. The distance from the center of the barrel wheel to circumference of the arbor wheel is $C - Ra$, and so:

$$\Sigma = Rb - (C - Ra) = Ra + Rb - C$$

Of importance is $Rb - \Sigma$, the space available for the screw hole and the ring of metal around it.

Finally, if there are n slots in the barrel wheel then n - 1 sections have the width $\O$ and

$\O' = 360° - (n - 1)\O$ is the width of the section with the locking boss.

For example, the other Breguet watch has $Rb = 1.86$, $Ra = 1.75$, $\partial = 15°$, and consequently for equal segments $\O = 72°$ and $C = 2.54$. The animation http://www.watkinsr.id.au/Animations/Animations-Equal-Segments/Animations-Breguet-Other-72-degrees.html was created using these measurements.

The above radiuses are the actual sizes of the wheels in millimetres. And the wheels in watch No.28 are only slightly larger, both having a radius of about 2.12 mm.

These small sizes have a serious consequence. In the actual design for the other watch the radius of the screw hole $Rh = 0.52$ mm, the radius of the screw head $Rs = 0.75$ mm, $\Sigma = 0.93$ mm and each slot is 0.95 mm deep. This leaves about 0.39 mm between the base of the slot and the hole for the screw, and 0.18 mm between the base of the slot and the shoulder for the screw. Figure 47 shows that, for the equal segments design above, the slots must be deeper. Slot 4 has been extended to the required depth, 1.07 mm, and slot 3 has not. Now the slots are 0.06 mm from the outside of the shoulder (yellow circle) and the metal between the slot and the hole for the screw is only about 0.27 mm wide.

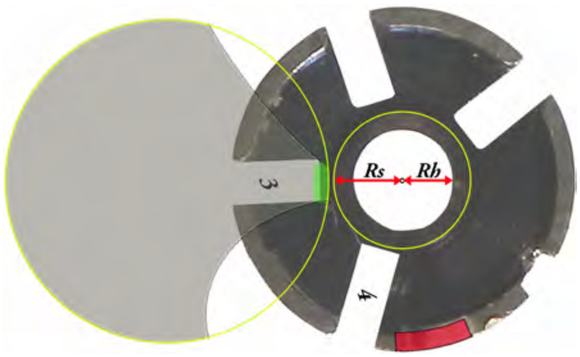

Figure 47

Clearly this has reached or passed the requirements for a satisfactory barrel wheel able to function without breaking.

Figure 45 represents one extreme, where the finger enters at the extremity of the next slot and ∂ is as large as possible. But, as in Figure 46, another center distance can be used.

In Figure 48, showing equal segments, the barrel wheel has been moved away from the arbor wheel so that $\boldsymbol{\partial}$ is smaller, but still wider than the end of the finger.

This figure is taken from the animation of this stop-work based on the original data, and it has a center distance of 2.68 mm. From this diagram we can measure $\boldsymbol{\partial} = 9^\circ$ and using the above formula:

$\boldsymbol{\mu} = (72 + 9)/2$ and $\boldsymbol{C} = \boldsymbol{Rb} \cos(\boldsymbol{\mu}) + \boldsymbol{Ra} \cos(\boldsymbol{\beta}) = 2.680570061$

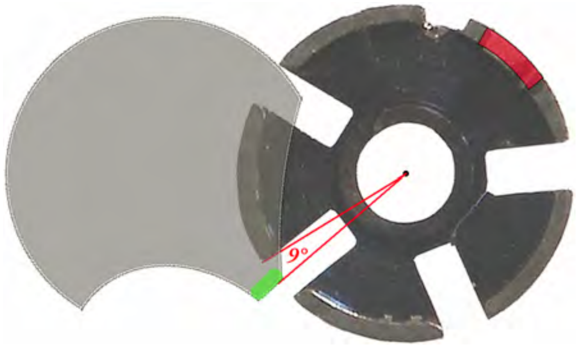

Figure 48

A rather nice confirmation that the formulae are correct and that this other watch functions with equal segments!

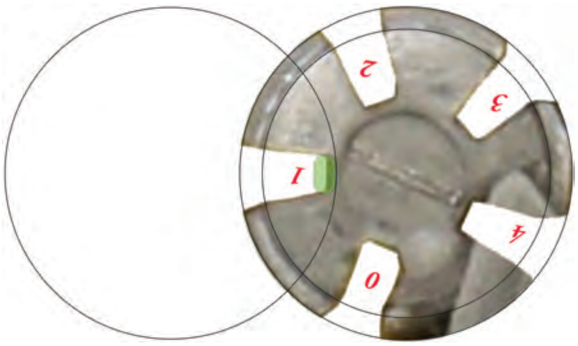

Figure 49

Unfortunately I do not know the actual dimensions of watch No. 28. The animations of it are based on the arbor and barrel wheels having the same diameter, but they are (not obviously) wrong. Figure 49 shows what is hidden: it appears that the finger overlaps the base of the slots, which is impossible.

The problem is that the photographs are mediocre and not perfectly perpendicular. Also, it is not clear what is the metal of the barrel wheel and what is shadow, and the wheels may have different diameters. Most importantly, reducing the actual diameter of the arbor wheel to 97% of the barrel wheel, from 2.12 mm to 2.06 mm (a change of 0.06 mm) appears to correct the error.

However, animations using the smaller arbor wheel fail completely, because the arbor wheel crown butts against the barrel wheel crown instead of entering a slot, Figure 50.

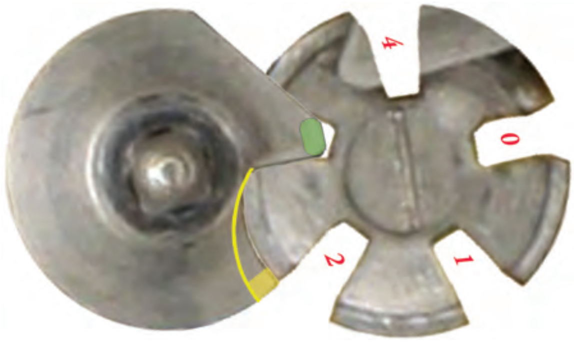

Figure 50

Here the finger is rotating the barrel wheel, but before the finger leaves the slot, the crown of the arbor wheel butts against the barrel wheel instead of entering the next slot. For this to work and wind the mainspring the crown of the arbor wheel has to be cut back, removing the yellow area, presumably by changing the cut-out to follow the yellow curve.

However, this illustration is also misleading.

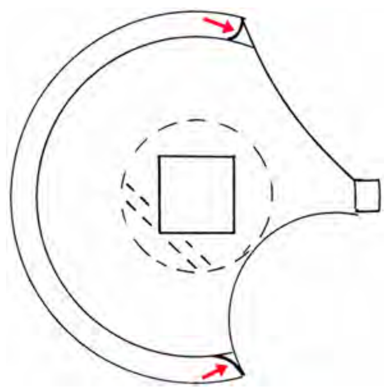

Figure 51

Figure 51 is a drawing of the arbor wheel from the other Breguet watch, and it shows that the ends of the crown are tapered in a curve. Also, in watch No. 28, the slots have opening mouths. These features, together with errors in the photographs, suggest that watch No. 28 may work with the smaller arbor wheel, because the tapered end of the crown can enter the slot by rotating the barrel wheel a little clockwise.

The following table gives examples showing some of the variations that can be produced by changing different dimensions; the entries are explained in the following notes. The ***∂*** and ***C*** entries in bold are calculated from the other values. ***Σ%*** is the slot depth as a percentage of the barrel wheel radius. Animations are available from http://www.watkinsr.id.au/Animations/.

Note	Type	***Ra***	***Rb***	***Ø***	***∂***	***C***	***µ***	***ß***	***Σ***	***Σ%***	***Ø'***
a	2-Pin	2	2.5	72	6.0	***3.18***	39.0	51.9	1.32	52.9	72
b	2-Pin	2	2.5	72	11.0	***2.99***	41.5	55.9	1.51	60.3	72
c	No 28	2.12	2.12	70	20.0	***3.00***	45.0	45.0	1.24	58.6	80
d	No 28	2.12	2.12	70	15	***3.13***	42.5	42.5	1.11	52.5	80
e	No 28	2.12	2.12	70	9.0	***3.27***	39.5	39.5	0.97	45.7	80
f	No 28	2.12	2.12	72	***17.8***	3.00	44.9	44.9	1.24	58.3	72
g	No 28	2.12	2.12	72	20.0	***2.95***	46.0	46.0	1.29	61.1	72
h	No 28	2.06	2.12	70	***16.3***	3.00	43.1	44.8	1.17	55.2	80
i	No 28	2.06	2.12	70	20.0	***2.91***	45.0	46.8	1.27	59.9	80
j	No 28	2.06	2.12	72	***14.3***	3.00	43.1	44.8	1.17	55.2	72
k	Other	1.75	1.86	67	***14.0***	2.68	40.5	43.7	0.93	50.0	92
l	Other	1.75	1.86	72	***9.0***	2.68	40.5	43.7	0.93	50.0	72
m	Other	1.75	1.86	72	15.0	***2.54***	43.5	47.0	1.07	57.5	72
n	***Ra*** > ***Rb***	1.96	1.86	66	15.0	***2.96***	40.5	38.1	0.86	46.4	96
o	***Ø*** > 72	1.75	1.86	74	***7.0***	2.68	40.5	43.7	0.93	50.0	64

Table of Designs

(a) This is the design described in Figure 22. In all the animations ***∂*** = 6°, even when the slot width is increased to 11°. In Figure 26 I assumed the theoretical ideal where the two radiuses meet at 90°. Here the angle is 180 - ***µ*** - ***ß*** = 89.13°, not a significant difference, but ***Ra*** and ***C*** have changed from 1.82 and 3.09 in Figure 22 to 2.0 and 3.18 respectively.

(b) If ***∂*** is increased to 11° then the center distance ***C*** must decrease and the slots must be deeper.

(c) These are my original estimates for the dimensions of No. 28 and are used to calculate ***C***.

(d) These are the values for Figure 38. In this table ***C*** *is the minimum value of the center distance for* ***w*** = ***∂***.

(e) If ***∂*** is decreased to the smallest value allowed by the size of the finger, then ***C*** must be increased to its *maximum value*.

(f) If the barrel wheel is equally divided with ***Ø*** =72°, and ***C*** is constant at the original value of 3.00, then ***∂*** changes to 17.8°.

With these parameters, the mainspring *cannot* be wound unless the arbor wheel crown is cut back; in the animations and Figure 52 there is a rectangular notch, but it could be tapered as in Figure 51.

Figure 52

However the mainspring *can* be unwound without any changes; the difference is caused by the asymmetry of the arbor wheel.

(g) Similarly, if ***∂*** is kept constant at 20°, then ***C*** decreases as expected. Again the mainspring cannot be wound without changing the arbor wheel crown.

(h) If the diameter of the arbor wheel is reduced to 97% of the diameter of the barrel wheel, and ***C*** is kept constant, then ***∂*** decreases to 16.3. Again the mainspring cannot be wound without changing the arbor wheel crown, in the animation by tapering it as in Figure 51, or by filing it back as in Figure 50.

Figure 50 is misleading, because it appears to suggest that ***∂*** = 0. But that figure shows the effect of the arbor wheel crown, and ***∂*** can only be determined if the wheels can continue to rotate until the finger starts entering the next slot.

(i) If the diameter of the arbor wheel is reduced to 97% and ***∂*** is kept constant at 20°, then ***C*** decreases; the difference is quite large, about 0.09 mm. There is no animation because the behaviour is the same as *(f)* to *(h)*.

(j) With equal segments, if the diameter of the arbor wheel is reduced to 97% and ***C*** is constant at the original value of 3.00, then ***∂*** changes to 14.3°. Again, the watch will wind only if the arbor wheel crown is cut back. The animations for *(h)*, *(i)* and *(j)* suggest that the arbor wheel in watch No. 28 is the same size as the barrel wheel.

(k) With the other watch we know ***Ra***, ***Rb*** and ***C***, and this entry calculates ***∂*** based on ***Ø*** = 67°. The slot depth ***∑*** is calculated to be 0.93 and the actual slots are 0.95. ***∂*** = 14° is sensible because it allows 1° of freedom.

(l) For equal segments and keeping ***Ra***, ***Rb*** and ***C*** constant, ***∂*** is calculated to be exactly 9°, confirming Figure 48.

(m) Alternatively, as in Figure 47, we can keep ***∂*** constant and calculate ***C***. In the actual barrel wheel, Figure 53, the slots are 0.95 mm deep and the screw fixing the wheel to the barrel sits on a shoulder that is 0.23 mm wide and 1.5 mm in diameter. So there is 0.16 mm between the base of the slots and the screw shoulder. With equal segments the finger moves 0.11 mm closer to the center of the barrel wheel and this reduces to 0.05 mm, unacceptably small. So we must conclude that any design which reduces ***C*** is probably not possible and reject (*b*), (*g*) and (*i*) above.

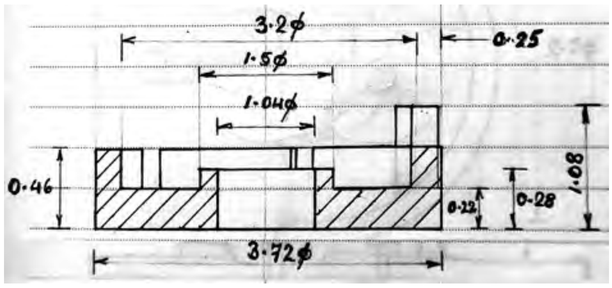

Figure 53

(n) What happens if the arbor wheel is larger than the barrel wheel? Using the other watch at (*k*) and changing ***Ra*** makes the center distance larger.

(o) Can ***Ø*** be greater than 72°? Yes. When ***Ø*** is increased to 74° then ***Ø'***, the width of the segment with the boss, reduces from 92° at (*j*) to 64°. This is large enough for the stop-work to function correctly.

Finally, this table confirms that Daniels was wrong when he stated that *"In order that [the boss] will not raise [the annulus] in the unwound position its segment is slightly longer than the other four ..."*. ***Ø'*** can be larger than, the same size as, or smaller than the other segments.

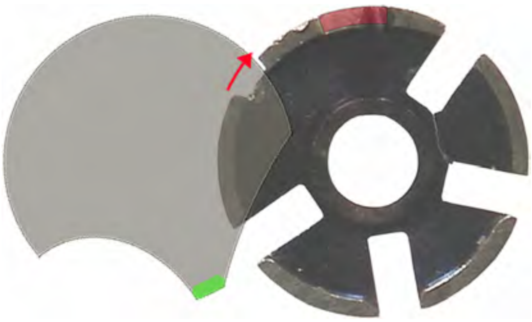

Figure 54

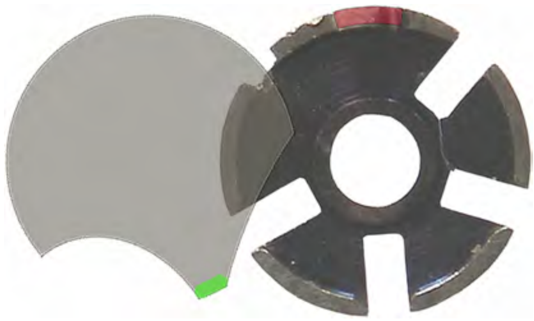

Figure 55

Breguet's design, as used in watch No. 28 and the other watch, has a serious fault.

Figure 54 shows the stop-work not long after winding has started. In this position the barrel wheel is free to rotate clockwise (red arrow) until the crowns butt against each other. Figure 55 shows this position.

Now, as the arbor wheel continues to rotate anti-clockwise, the finger will butt against the crown of the barrel wheel, Figure 56.

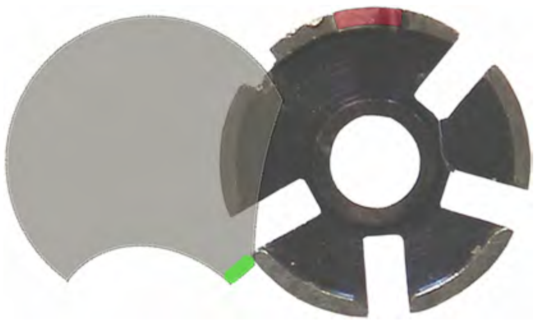

Figure 56

At this point the weight is free and will continue to force the arbor wheel to rotate, resulting in catastrophic damage.

Exactly the same problem occurs with watch No. 28.

The other breguet watch also shows this fault in a different position. In Figure 57 the arbor wheel has rotated anti-clockwise until the finger is about to leave a slot. At this point the barrel wheel can rotate so that the crown of the arbor wheel will butt against the barrel wheel crown, Figure 58. However, this fault can be avoided by making the arbor wheel crown longer, so that it has already entered the slot in Figure 57. It does not occur in watch No. 28.

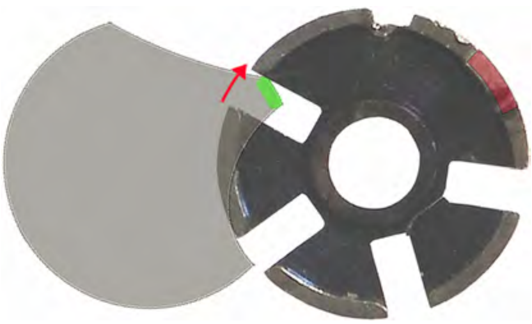

Figure 57

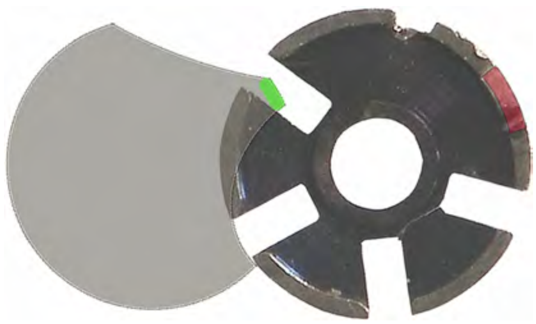

Figure 58

The only solution to these problems is to ensure that the barrel wheel is held in position by friction. The friction must be enough to avoid accidental movement, but still allow the arbor wheel to turn it easily.

However, this fault can be avoided by reducing $\boldsymbol{\partial}$ to its minimum value. In Figure 59 the top arrow shows the possible rotation of the barrel wheel and the bottom, identical arrow shows how this movement affects the finger; unlike Figure 56, the finger cannot move to a position relative to the barrel wheel where it is blocked by the barrel wheel crown.

Similarly, Figure 60, unlike Figure 58, shows that the movement of the barrel wheel cannot cause the two crowns to butt against each other.

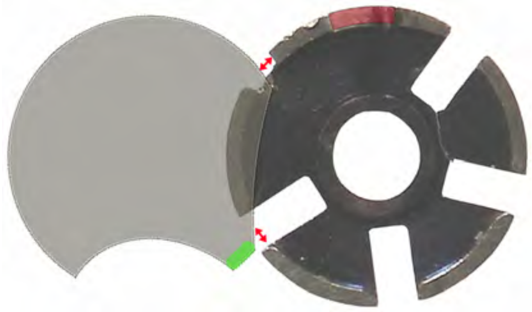

Figure 59

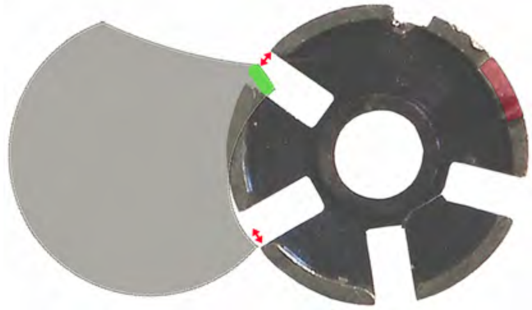

Figure 60

Animations of these faults are available from http://www.watkinsr.id.au/Animations/Animations-Meditations/.

The restoration of an antique watch should be done without changing the design, but the fault can be removed by changing either the arbor wheel or the barrel wheel. Because this is a repair and the barrel wheel has been screwed to the barrel, we cannot change the center distance ***C***. The two options are:

(a) Change the diameter of the arbor wheel: We know ***C***, ***Rb***, ***Ø*** and ***∂***, and hence μ. Using Figure 46, calculate ***y*** and ***Cb*** and hence ***Ca*** = ***C*** - ***Cb***. Then Ra = $\sqrt{(y^2 + Ca^2)}$. In the other watch, the new value of ***Ra*** is reduced from 1.75 mm to 1.67 mm.

(b) Change the diameter of the barrel wheel. This requires a different method. In Figure 61 we know ***Ra*** = 1.75, ***C*** = 2.68, ***Ø*** = 67, ***∂*** = 9, and μ = 38. Using the law of sines:

$Ra/\sin(\mu) = C/\sin(\gamma) = Rb/\sin(\beta)$

Using the first two equalities:

$\sin(\gamma) = C \sin(\mu)/Ra$

Two values of γ satisfy this equation, for the two triangles ***a b c*** and ***a b d***. The one we want is for the former triangle, $\gamma = 109.47^\circ$ (With the other triangle the crown of the barrel wheel would have go through the barrel arbor!) . From this value $\beta = 180 - 38 - 109.47 = 32.53^\circ$.

Now using the second and third equalities:

$Rb = C \sin(\gamma)/ \sin(\beta)$ = 1.53 mm when it was 1.86 mm.

Figure 62 shows the result, where the shades parts are the slots.

The above assumes ***Ø*** = 67° for the smaller barrel wheel. This is correct. But assuming the same cutter is used to make the slots, then the angular width of the slots increases from 16° to 18° and the angular width of the segments decreases from 51° to 49°.

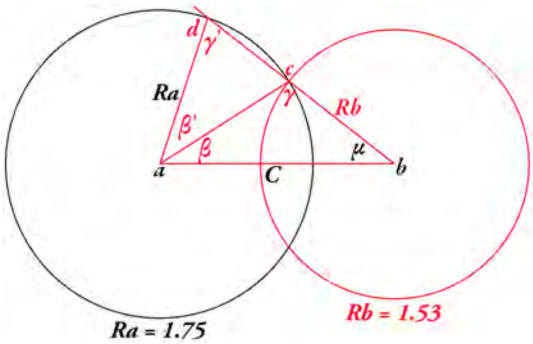

Figure 61

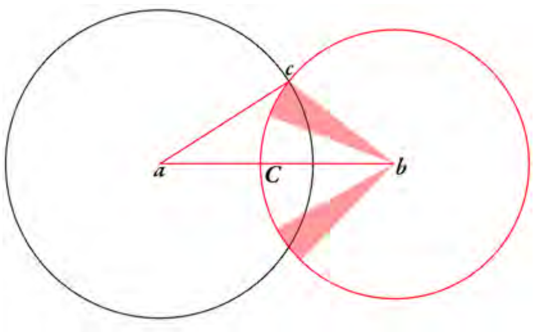

Figure 62

In addition to this fault, there is another interesting question:

Is there any situation when the force of the mainspring acts on the stop-work?

Most of the time there should be no force. During winding and unwinding the click-work of the winding train should counteract the torque of the mainspring. But when the mainspring has run down there remains the torque produced by the set-up of the spring, and this must be counteracted by the stop-work. To put this in context, the following table gives the approximate dimensions of Watch No. 28 and the actual dimensions of the other watch in millimetres.

		Watch No. 28	Other watch
Arbor wheel	Diameter	4.24	3.50
	Crown width	0.40?	0.40
	Finger size	0.4 x 0.47?	0.40 x 0.40?
	Base width	0.21	0.21
Barrel wheel	Diameter	4.24	3.72
	Crown width	0.31	0.25
	Slot width	0.48	0.46
	Slot depth	1.20	0.95
	Base width	0.21	0.22
Center distance		3.01	2.68

Stop-Work Dimensions

In Figure 63 the barrel is trying to rotate anti-clockwise to run the watch, but its motion is impeded by the finger, which must be strong enough to withstand the torque. Figure 64 is the same position in the 2-pin design. Here the green pin transmits the torque to the arbor wheel. This is a weak design, because the small pin must be screwed and/or riveted to the base of the barrel wheel.

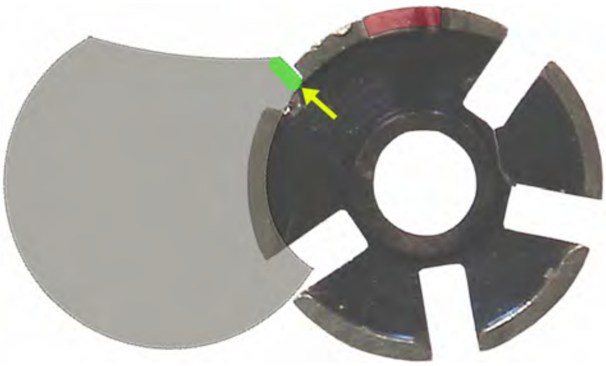

Figure 63

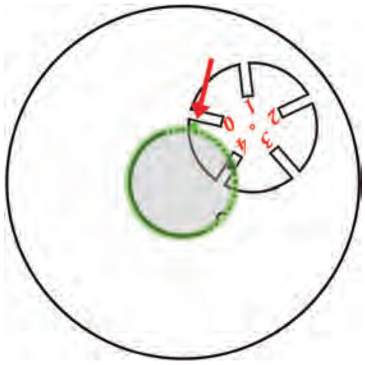

Figure 64

In Figure 65 the torque is transmitted from the crown of the barrel wheel to the base of the arbor wheel. Clearly this is the best design from that perspective.

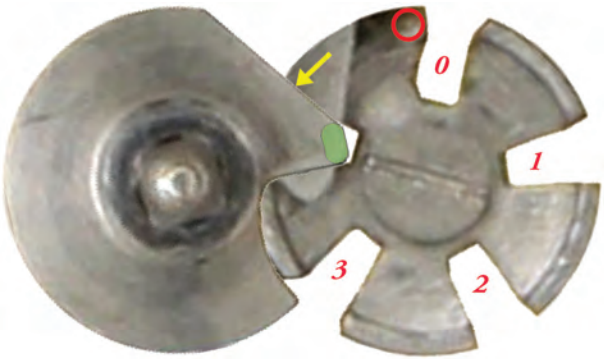

Figure 65

Finally, from photographs of watches it appears that Breguet may have made stop-work with 6 slots. In principle, such stop-work uses **Ø** = 60° and its design is similar to the 5-slot designs described above. The following table gives three examples for a 6-slot design:

Ra	*Rb*	*Ø*	*∂*	*C*	*µ*	*ß*	*∑*	*∑%*	*Ø'*
2	2	60	15.0	3.17	37.5	37.5	0.83	41.33	60
2	2	55	15.0	3.28	35.0	35.0	0.72	36.17	85
2	2	50	9	3.48	29.5	29.5	0.52	25.93	110

Table of 6-slot designs

5: God and Chickens

"When I asked you to choose a title for this section, I did not expect something so bizarre. What do you mean?"

"Which came first, the chicken or the egg?"

"What on Earth does that have to do with Breguet?"

"You will find out. I have read what you are going to write."

"That is cheating! Go away. I do not like someone looking over my shoulder."

Although I, and hopefully you, now understand how Breguet's stop-work functions, there is another very important question:

> *How did Breguet and his workmen* ***make*** *the stop-work?*

But first we need to know the answer to the more general question:

> *How did Breguet and his workmen make watches?*

The only information I have seen comes from "The Art of Breguet" by Daniels, which has a section on workshop practice. Although Daniels does not explain how he knew, he is certain that Breguet made watches using the same methods as used by other 18th century watchmakers, such as Berthoud, Auch and Vigniaux. And he states:

> *"since no two workmen think alike, no two components are found to be identical."*

> *"Breguet needed to lay down rules for construction to enable his workmen to produce the requisite style of work without the tedium of making identical pieces to pattern."*

(Daniels also notes that "In the author's experience it takes one man some nine months to make one watch including the case, dial and hands." Although complex watches may take that long, ordinary watches could made in about one month.)

Actually we can go further. In "Origins" (Watkins, 2016, page 372) I point out that I read somewhere that Breguet continually changed his designs to incorporate improvements. But the wings on the weight arms of Breguet's self-winding watches are all unique and the differences seem to be arbitrary rather than a process of improvement. That is, his watches were *deliberately* unique.

Unfortunately, with the exception of "The Art of Breguet" none of the books I have seen provide any useful technical information. They only have general views of movements that show none of the important features, and general statements that do not explain the mechanisms. And with regard to self-winding watches, the explanation by Daniels is wrong (see "Origins", page 366).

As a result we have information about only two watches and a 2-pin design that may never have been used. But these three designs are significantly different and they suggest that the stop-work on *all* self-winding watches is probably different, reflecting the way in which they were made so that "*no two components are found to be identical.*"

In this context pentagon creation, Figure 27, is irrelevant, because it is likely that none of the self-winding watches have equal segments. And for the same reason we can also assume that Breguet did not use a division plate to divide the barrel wheel into 5. (Although irrelevant, an excellent discussion on making such plates can be found in Weiss, 1982, pages 129-151.) That is, the stop-work was made by hand with minimal use of special tools.

However, the workmanship in Breguet's watches is obviously excellent. This suggests that the barrel wheel in the other watch might be original. And it suggests that the barrel wheel in watch No. 28 is a replacement made by an inferior watchmaker; certainly the wheels in Figures 63 and 65 show a significant difference in quality. So it is likely that both watches have failed in the past. Although a sample of two is not enough, it does suggest that Breguet's design was far from perfect.

So how was the stop-work made? The following explanation is possible.

Note that the watchmaker does not have to measure anything.

(1) *The blanks:* (Figure 66) Take a piece of steel rod, punch centers in both ends and drill a hole through the center for the screw. Cut off the rod. Put the blank on a turning arbor and, with a bow, turn the outside concentric and make a recess to form the crown, leaving a boss in the center that is less than half of the radius of the rod.

Repeat the process to make the arbor wheel blank, but do not leave a boss in the center. Drill a large hole through the center and punch out the hole into a square.

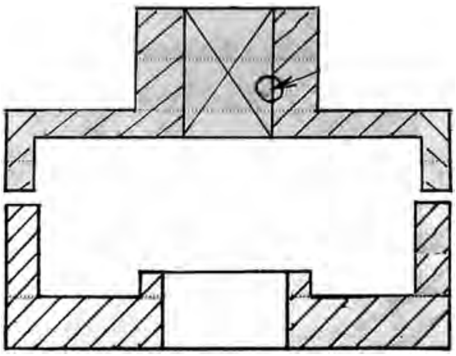

Figure 66

At this point the crowns are high enough to form the arbor wheel finger and the barrel wheel locking boss and they will have to be filed back later.

Also, we have the first chicken and egg problem. The arbor wheel fits on a square on the barrel arbor, and the height of that square depends on the height of the barrel wheel crown that the arbor wheel must fit over. But the wheels have not been finished and this height has not yet been determined. Also, the square cannot be too low, or else all of the crown of the arbor wheel would be removed. Presumably the height of the square can be adjusted later.

(2) *Cut slot 1:* (Figure 67) Mount the barrel wheel (preferably in a wheel cutting engine) and cut one slot so that the depth of the slot is about ½ the radius of the wheel and the base of the slot is a little away from the central boss for the screw.

There is a magic trick here. From the formula for Σ given earlier:

$$C = Ra + Rb - \Sigma$$

That is, the slot must be about the right depth to get a suitable value of C, and $\Sigma = Rb/2$ is a good approximation. It is a trick, because nothing is being made to measurements and the slot depth is guessed.

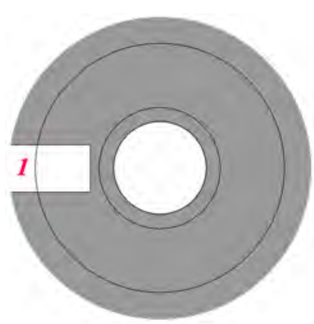

Figure 67

(3) *Make the finger:* (Figure 68) Cut back the arbor wheel crown and base sufficiently for the wheel to be placed over the barrel wheel and shape the finger so that it will enter the slot. File back the crown of the arbor wheel enough for the finger to butt against the barrel wheel crown.

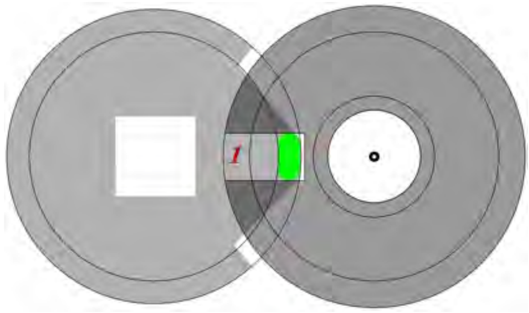

Figure 68

The white cut-out shows the area of the arbor wheel base that has been removed. The locking boss on the barrel wheel crown will be placed at about the top of the barrel wheel in this diagram, so some of the crown around the slot could be filed down a bit. Note that the barrel arbor square, and the arbor wheel square hole in Figure 66, are long enough to put the unfinished wheels together.

(4) *The center distance:* (Figure 68) Using the barrel and barrel arbor without the mainspring, put the arbor wheel on the barrel arbor square and position the barrel wheel so that the finger is near the base of the slot. Mark the barrel wheel center on the barrel, drill and tap the hole for the screw and fix the barrel wheel to the barrel.

(5) *Cut slot 2:* (Figure 69) Put the arbor wheel on the barrel arbor and rotate both wheels until the finger starts leaving slot 1. Mark the point where the two crowns meet (red arrow) and file the arbor wheel base and crown back to the mark. Using the mark on the barrel wheel cut slot 2.

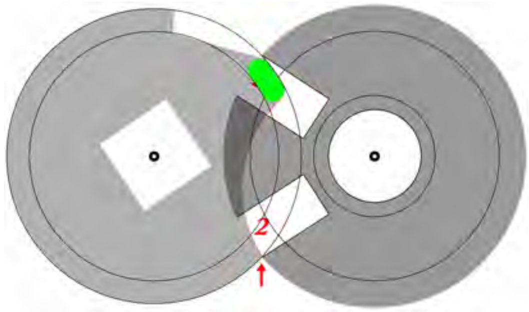

Figure 69

Slot 2 can be cut anywhere that allows the arbor wheel finger and crown to enter it. But this position is the easiest to make and replicate for the other slots.

I have not yet made the annulus. It will be about the diameter of the arbor wheel, but possibly a bit smaller or a bit larger. The graceful curve of the arbor wheel allows for this variation.

(6) *Cut slot 0:* (Figure 70) Rotate the barrel and arbor wheels so that the finger is in slot 1. Mark the point where the two crowns meet (red arrow) and file the arbor wheel base and crown back to the mark. Using the mark on the barrel wheel cut slot 0.

Slot 0 could have been cut first. I decided to cut slot 0 to its full depth and arrange the locking boss to suit this design.

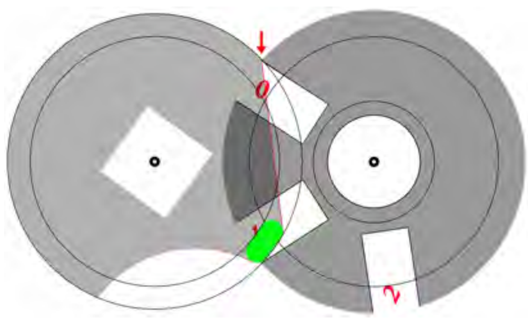

Figure 70

(7) *Cut slots 3 and 4:* (Figure 71) Cutting these slots is the same as cutting slot 2 in step (5).

(8) *Adjust the crowns and the barrel arbor square:* Until this step the crowns and the finger have been made too high and cut back just sufficiently to put the wheels together so that the positions of the slots can be determined.

Now all three and the barrel arbor square are filed back until the arbor wheel sits over the barrel wheel with a little freedom between the crowns and the bases of the wheels, and the finger reached the bottom of the slots, with a little freedom between it and the barrel. In this step the barrel wheel crown between slots 0 and 4 is not touched.

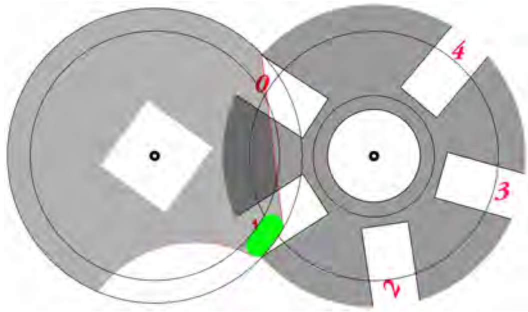

Figure 71

(9) *Make the locking boss:* (Figure 72) In my stop-work, the barrel wheel crown between slots 0 and 4 has four steps. The green part ***a*** under the arbor wheel is filed down to the same height as the other crowns. The yellow part ***b*** is filed level with the base of the arbor wheel; this part stops unwinding and absorbs the set-up torque of the mainspring. The red part ***c*** is the boss that lifts the annulus and locks the weight when the mainspring is fully wound; it is higher than the base of the arbor wheel. And the green part ***d*** is also filed down to the level of the other crowns so that it can, if necessary, fit under the arbor wheel base.

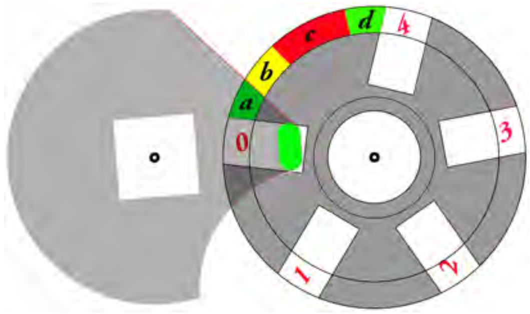

Figure 72

(10) Make the annulus: (Figure 73) Here is another chicken and egg problem. The annulus is a piece of steel shaped, from right to left, to form a foot, a spring, the annulus, and a thicker part used to raise the weight locking system. The foot must be the correct height to ensure the annulus is horizontal, but that height is not known until the barrel and arbor wheels have been finished.

Figure 73

Animations of this design are available from http://www.watkinsr.id.au/Animations/Animations-Meditations/.

In addition to 18th century tools, my virtual watchmaker's bench has some modern measuring tools. The following table gives the measurements in millimetres for the above design:

Ra	***Rb***	***w***	***C***	***∂***	***µ***	***Ø***	***ß***	***Σ***	***Σ%***
1.95	2.00	18.0	2.91	18.0	***41.9***	***65.75***	***43.2***	***1.04***	***52.0***

My Hypothetical Design

The steps used to make it mean that ***Ra*** (step 1), ***Rb*** (step 1), ***w*** (step 2), ***C*** (step 4) and ***∂*** (step 5) were chosen by the watchmaker. At no stage did the watchmaker know or need to know ***µ***, ***Ø***, ***ß***, ***Σ*** and ***Σ%***. In this design ***Ø'***, the width of the section containing the slot, is 97°.

Step 4 is very important because a small change in ***C***, caused by planting the barrel wheel a little closer or further away, will significantly change other dimensions. If the watchmaker made ***C*** about

0.14 mm larger (3.05 mm) and kept the other 3 dimensions the same, then ***Ø*** = 59.75° and ***Ø'*** = 121°. Actually this value of ***C*** makes 6 segments possible because ***Ø'*** is then about 61° which should be enough for the locking boss.

Step 5 is also very important. The watchmaker can cut slot 2 anywhere that allows the arbor wheel finger and crown to enter it, and ***Ø*** depends on this choice. For example, if the second slot is cut so that the finger enters the middle of it, then ∂ = 13.5° and ***Ø*** changes to 70.25°, but μ does not change because it is either (65.75 + 18.0)/2 or (70.25 + 13.5)/2; the formulae developed at Figure 46 show that μ only depends on ***Ra***, ***Rb*** and ***C***. However the design of the barrel wheel changes significantly and ***Ø'*** drops to 79.0°.

Finally, the design allows the mainspring to wind 3.98 turns.

This virtual stop-work made on my virtual watchmaker's bench is theoretically correct, but it is not practical because there is no freedom for the parts to operate satisfactorily. In reality the watchmaker must make the finger a little smaller (or the slots a little wider) and provide some freedom for the finger and arbor wheel crown to enter the slots easily. The latter could be done by making the slots a little wider, w = 19°, and leaving everything else the same; in particular ∂ remains at 18°. Or ∂ could be reduced to 17°, which would increase ***Ø*** by 1°.

Also there must be freedom between the bottoms of the slots and the finger, so when the barrel wheel is planted at step (4) it should be positioned as in Figure 68, which means that Σ is less than the slot depth. In my design Σ = 1.04 mm but the slots are 1.07 mm deep, allowing a clearance of 0.03 mm.

My method for making the stop-work required a magical trick at step (2). This is actually an important "chicken and egg" problem. My method of cutting a slot before planting the barrel wheel is based on the following argument derived from the formulae at Figure 46:

If we know ***Ra***, ***Rb*** and the slot depth Σ then we know ***C***:

$$C = Ra + Rb - \Sigma$$

If we know ***C*** then we know ***Cb***:

$$Cb = (Rb^2 - Ra^2 + C^2)/2C$$

$$= (Rb^2 - Ra^2 + (Ra + Rb - \Sigma)^2)/2(Ra + Rb - \Sigma)$$

And if we know ***Cb*** then we know μ:

$$\mu = \mathrm{acos}(Cb/Rb)$$

$$\mu = \mathrm{acos}([(Rb^2 - Ra^2 + (Ra + Rb - \Sigma)^2)/2(Ra + Rb - \Sigma)]/Rb)$$

Fortunately we can use a spreadsheet to calculate this rather than doing the arithmetic by hand.

The following table gives some designs based on knowing ***Ra***, ***Rb*** and Σ. In each pair one row is based on the magic trick Σ = ***Rb***/2.

The most important feature of this table is that we know μ = ***Ø*** + ∂, but ***Ø*** and ∂ are separately unknown, and consequently ***Ø'*** is also unknown. And w, the slot width, is unknown. The above table gives the theoretical values and in practice the watchmaker could use ∂ = w - 1 to allow freedom, and then ***Ø*** = μ - w + 1. But as the watchmaker measures nothing this relationship is meaningless.

This table and the previous table of designs show that Σ = ***Rb***/2 is a sensible approximation.

	Ra	***Rb***	***C***	***μ***	***Cb***	***Σ***	***Σ%***	***2μ***
My design	1.95	2.00	2.91	41.88	1.49	1.04	51.97	83.75
	1.95	2.00	2.95	41.04	1.51	1.00	50.00	82.08
Other Breguet Watch	1.75	1.86	2.54	43.55	1.35	1.07	57.53	87.09
	1.75	1.86	2.68	40.51	1.41	0.93	50.00	81.03
Breguet No. 28	2.13	2.13	3.00	45.10	1.50	1.25	58.82	90.20
	2.13	2.13	3.19	41.41	1.59	1.06	50.00	82.82
Other with Ra > Rb	1.96	1.86	2.82	43.81	1.34	1.00	53.76	87.62
	1.96	1.86	2.89	42.15	1.38	0.93	50.00	84.31
Breguet 2-pin	2.00	2.50	3.18	38.97	1.94	1.32	52.80	77.93
	2.00	2.50	3.25	37.96	1.97	1.25	50.00	75.92

Designs Based on Σ

There is an alternative method to construct the stop-work. Instead of using step (4) above, the center distance can be determined from ***μ***. If we know ***Ra***, ***Rb*** and ***μ*** then ***C*** can be determined by reversing the formulae given above, starting with calculating ***Cb***, then ***C*** and finally ***Σ***; again this is of interest to us but irrelevant for the watchmaker who measures nothing.

From the two tables ***μ*** varies between about 40° and 45°. And so the first four steps can be changed to:

(1) *Make the blanks.*

(2) *Fix the center distance:* Put the arbor wheel on the barrel arbor and the barrel wheel on the barrel. Move the barrel wheel until the angle between the line of centers and the intersection of the crowns is about 45°, or preferably a little less, and mark the barrel wheel center on the barrel.

(3) *Cut slot 1:* During step 2 also mark where the arbor wheel crown crosses the line of centers. Cut the slot a little deeper than the mark.

(4) *Make the finger:* Use the method described above.

(5) *Cut the remaining slots:* Use the method described above.

Judging 45° is easy, but this method is probably a little harder because the crowns have not been cut back, and balancing the arbor wheel over the barrel wheel might be difficult.

Actually, creating stop-work with equal segments is not only possible but easier:

(1) *Make the blanks.*

(2) *Cut 2 slots:* Using a wheel cutting engine, cut 2 slots 72° apart and as deep as practical.

(3) *Fix the center distance:* (Figure 74) Put the arbor wheel on the barrel arbor and the barrel wheel on the barrel so that the crown of the arbor wheel enters the 2 slots and does not go closer to the center of the barrel wheel than the base of the slots (yellow circle).

Mark the barrel wheel center on the barrel and plant the barrel wheel.

(4) *Make the finger:* Use the method described above.

(5) *Cut the remaining slots:* Using a wheel cutting engine, cut 3 slots 72° apart. Then finish the wheels as described above.

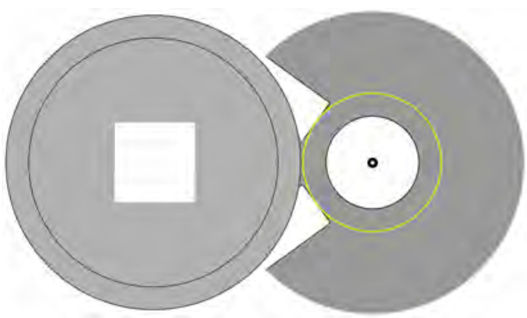

Figure 74

There is some flexibility in Step (3), because ***C*** can vary a little. In Figure 74 I moved the barrel wheel closer to the arbor wheel so that ***C*** = 2.89 mm and ***∂*** = 12.61°. But I could have used the original value of ***C*** = 2.91 mm and have ***∂*** = 11.77. Or I could use ***C*** = 2.98 mm, which gives the smallest ***∂*** = 9°. So the watchmaker has nearly 0.1 mm of freedom in planting the barrel wheel.

Not only is this method easier, but it uses values of ***∂*** that are less likely to have the faults I have described earlier.

The three methods above illustrate the chicken and egg problem. The main dimension that the watchmaker must set is ***C***, and I have done this in three different ways:

(a) Define ***Σ*** and set ***C*** from it.

(b) Define ***C***.

(c) Define ***Ø*** and set ***C*** from it.

Although ***w*** is defined in all three methods, and thus the maximum and minimum values of ***∂***, the width of the slots is arbitrary.

It is actually quite hard to avoid making 5 slots. Unless the stop-work is made using the equal segments method above, 6 slot designs (as in the table on page 28) require more thought and care.

Finally, Figure 75 shows stop-work wheels at approximately their actual size.

Figure 75

6: Breguet and Mathematics

God has a home theatre in which he watches his universes, and the second best seat is on the right hand of God. This is because God is left-handed and the person on his right gets more popcorn than the others. To decide who had the honour of sitting there, Mrs God (also known has "she who must be obeyed") chose people by a cooking competition.

The first to win was Simon, who made a delicious roast beef with Yorkshire pudding. Then Joseph had his turn as a result of a wonderful coq au vin and crepes suzette, followed by Marco with gnocchi in a superb tomato, basil and pine nut sauce. Much to everyone's surprise, Richard earned the right to sit beside God by producing a spanakopita to die for.

So there was happiness and contentment, even though God was getting overweight.

Unfortunately Breguet was not a good cook and he was relegated to the back row. (Having read a few books about Breguet I have the impression that Breguet is revered as the greatest watchmaker who ever lived. Indeed, I suspect that some people believe that he must sit beside God. As you now know, these writers are wrong.)

"A little while ago you accused me of being bizarre. But what in Heaven has my home theatre and cooking have to do with mathematics?"

"Nothing."

"So why write about it?"

"I started writing this article when I asked myself: *Did Breguet use mathematics to design his stop-work?* I have already answered that question, so I don't have anything to write."

"Perhaps Breguet used mathematics for other parts of watches. You need to consider that possibility."

First, Breguet's biographies contain statements such as:

> *... Breguet followed a course of mathematics given by the Abbé Marie ... under his excellent tuition, the young man's talents were fostered to the full.* (Chapuis & Jaquet, 1956.)
>
> *During his apprenticeship he attended evening classes at the Collège Mazarin where he learned mathematics.* (George Daniels, 1975, who suggests that this was before 1768.)
>
> *... but he certainly studied mathematics at the Collège Mazarin under Abbé Marie.* (Antiquorum, 1991. (Antiquorum implies this happened after he had completed his apprenticeship.)
>
> *With Abbé Marie, Breguet first of all studied mathematics through several years of lectures and individual lessons.* (Emmanuel Breguet, 1997.)

The earliest reference to mathematics and the Abbé Marie that I have read, is in an 1849 book by Pierre Dubois.

I am assuming *mathematics* in this context means something much more sophisticated than arithmetic and geometry. Surely it should include algebra and trigonometry? Perhaps even some basic calculus? And surely to stress that Breguet had such knowledge should imply that he actually used it?

Books have been written that use advanced mathematics and calculus to explain watches, but these, mainly 20th century books are academic treatises of little or no use to watchmakers.

The 18th century watchmaker had no need for anything more than simple arithmetic and perhaps a little geometry. Berthoud, Auch and Vigniaux manage very well without sophisticated calculations, and Thomas Hatton teaches the necessary mechanical skills with just these two aspects of elementary calculation.

The reason is simple. The inadequate measuring tools and the inability to machine parts accurately meant that calculating precise dimensions was a waste of time. To know that my spreadsheet calculates a center distance as 2.910555015 mm is as absurd today as it was then, and even rounding it to 2.91 mm is optimistic.

Three examples:

First, in 1752 Camus provided the theory of epicyclic gears for the teeth of wheels. But watchmakers never used correct teeth and leaves, because they were shaped by hand filing them, and it is impossible to make epicyclic teeth without special cutters. And it is almost impossible to make the cutters; as I note in "Origins" (Watkins, 2016, pages 81-82) even in the mid 20th century epicyclic teeth were not used. So watchmakers used thumbs and bay leaves.

Second, the theory of springs dates back to the 17th century and has been developed since then. But it was, and to some extent is, useless for watchmakers. In 1780 Blakey described the process of making springs. In addition to the variable quality of the steel used, spring making was an imprecise art. Knowing the behaviour of a theoretically ideal spring was useless, because real springs behaved differently and erratically. For example, the theory of fusees was also useless, because fusees had to be "equalised", shaped, to match the variable strength of the mainspring; and whenever a mainspring was replaced the fusee should be re-equalised. (Actually, it is likely that most fusees were never equalised. The vast majority of watches with fusees were ordinary watches that did not justify the work, and even a fusee that has not been equalised significantly smooths the power going to the escapement.) And as late as 1893, because it was impossible to make balance springs of exactly the same size, weight, shape and quality of steel, balances and balance springs had to be matched to each other (Houriet, 1893). Even in the 20th century watch adjusting was an art and not a science, with the theoretical shapes for balance spring overcoils only provided a starting point for the manipulation necessary as part of adjusting.

Third, the mathematical analysis of many watch parts, which are asymmetrical, is impossible. For example, repeaters are designed and made on the basis of practical experience and not theory; see Crespe (1804) and Watkins (2011). I mention repeaters because they were the most popular complication used by Breguet. Daniels (1975) includes photographs and information for about 257 watches, and of these 37.7% are repeaters, 14.4% perpetuelles, 13.2% "simple" watches, and 10.1% souscription watches. The number of repeaters is larger, because perpetuelles and other watches included repeater mechanisms, and probably more than 40% had this complication.

Added to these problems was the inaccuracy of the methods used. In addition to my previous quotes (see page 28 above), Daniels wrote:

> *When all is correct* [the wheels and pinions carefully matched together] *the wheels are fitted to the plate by filling the original holes, marking off the new, correct positions and re-drilling the holes.*

This was standard practice in the 18th century.

Indeed, *"making identical pieces to pattern"* was impossible, although it would produce similar watches. Consequently, much of the watchmaker's training was learning methods to *overcome inaccuracy*. And the higher the quality of the watch the more work had to be done.

However, I think even similarity (as opposed to interchangeability) was irrelevant to Breguet. As I have suggested in "Origins" (page 372) it is likely that he deliberately created unique watches to make them more attractive to potential buyers.

In addition, Daniels (1975, pages 49-56) provides facsimiles of pages from Breguet's work books. These include pages with disorganised calculations that involve only arithmetic; there no algebra, geometry or other mathematics. As Daniels writes:

> *His sketches show that he had a somewhat muddled and untidy mind where details were concerned.*

All this is consistent with his self-winding watch stop-work and supports my suggested methods for making it. Trigonometry and algebra were very useful for me (and you?) to understand the stop-work, but mindless animations were more useful. However, constructing them requires no mathematics, just the normal 18th century practices. Indeed, mathematics would be a liability, because it only enables us to define precise dimensions that would be almost impossible to make in practice. I should note that, with one exception, the mathematics in this article has no practical use; the exception is $\boldsymbol{Nr} = \boldsymbol{Ns} + 2\boldsymbol{Np}$.

My opinion, that stop-work was *made* not *designed*, is based on only 2 watches and a design that may never have been used. So:

> *What stop-work is used in other watches?*

I suspect we will never know, because it is unlikely that the owners of Breguet's self-winding watches will allow them to be disassembled, photographed and measured. But I will make a prediction:

> *I believe that they will all be different, and any with equal segments will be accidents of manufacturing, not planned constructions.*

Of course there are restrictions on what can be done with two wheels, and the stop-work in some watches may be very similar, apparently identical. But again, I believe these similarities will be accidental and not deliberate. There is plenty of scope for variations, by changing some or all of ***Ra***, ***Rb***, ***C***, ***w***, ***Ø*** and ***∂***, and changes are inevitable when using 18th century techniques.

One point supports this view. The two watches examined above were made circa 1791 and circa 1810, nearly 20 years apart. But they both have the same fault, described at Figures 54 - 58. This suggests that Breguet never analysed the design mathematically and possibly did not know that the fault existed.

So what did Breguet use his knowledge of mathematics for? Probably, like me, he didn't use it. (50 years ago I studied mathematics for three years at university, but afterwards I had no use for it. So over the ensuing years I forgot nearly everything.) Breguet's inventions do not require mathematics, and his involvement with the Chappe telegraph was as a consultant on the "intricate system of pulleys and ropes" that would have required, at most, the same arithmetical skills as needed for watch trains. But perhaps his training sharpened his logic skills.

7: Explanation

My teleological argument is actually based on personal, irrational experiences.

My watchmaker God is derived from William Paley (1802). At the beginning of the first chapter of his book "Natural Theology" he argued for a such a creator:

> *In crossing a heath, suppose I pitched my foot against a stone, and were asked how the stone came to be there; I might possibly answer, that, for anything I knew to the contrary, it had lain there forever: nor would it perhaps be very easy to show the absurdity of this answer. But suppose I had found a watch upon the ground, and it should be inquired how the watch happened to be in that place; I should hardly think of the answer I had before given, that for anything I knew, the watch might have always been there. ... There must have existed, at some time, and at some place or other, an artificer or artificers, who formed [the watch] for the purpose which we find it actually to answer; who comprehended its construction, and designed its use. ... Every indication of contrivance, every manifestation of design, which existed in the watch, exists in the works of nature; with the difference, on the side of nature, of being greater or more, and that in a degree which exceeds all computation.*

And this reasoning is accompanied by a plate showing watch parts including a *contrite wheel*, although Paley does not explain why this wheel is remorseful.

An important consequence of this argument is it implies that the creator, or intelligent designer, of the watch or of the universe must be outside that creation.

As I suspect that no more than one person in one hundred million has ever read the rest of the book, the second chapter may come as a surprise:

> *Suppose, in the next place, that the person who found the watch, should after some time, discover, that, in addition to all the properties which he had hitherto observed in it, it possessed the unexpected property of producing, in the course of its movement, another watch like itself, (the thing is conceivable;) that it contained within it a mechanism, a system of parts, a mould for instance, or a complex adjustment of lathes, files, and other tools, evidently and separately calculated for this purpose ...*

The crudity of the analogy is obvious. Our universe is so bizarre that I find it hard to believe that an intelligent deity could have created it. Equally, it is so absurd I cannot imagine it was created by evolution.

It should be obvious that my God is an imaginary deity. Any similarity to Christian, Hindu, Muslim and other Gods and their universes is purely accidental.

Mrs God is based on a painting in Jaquet and Chapuis (1953, Plate XXI), Figure 76.

Figure 76

The firmament comes from a strange optical illusion. One night I looked outside and saw some lights shining quite brightly. My first thought was that they were stars or planets, but they appeared to be too close to each other. So I decided that they must be street lights. Whatever they were, they were a long way away. Next morning I looked in that direction and found there was nothing but the sky. That night I looked again, and there they were, distant lights that must be stars and could not be street lights, as Google maps confirmed. But stars should not be there. The riddle was solved the next evening. My next door neighbour is not a gardener. So he removed all the branches of a small tree, leaving a slender, five-meter trunk, and festooned it with fairy lights. As with Mrs God's firmament, the lights were much closer than I had imagined.

Any similarity between Richard, Joseph, Marco and Simon and any humans is also purely accidental.

I want to thank Anthony Randall for providing the photographs and measurements of the other Breguet self-winding watch.

Many of the diagrams have been created using http://geargenerator.com. The gears use involute teeth and not the epicycloid teeth used in watchmaking. That site animates the gears.

8: Animation

The animations are essential to my understanding of Breguet's stop-work. I found it impossible to predict the behaviour of stop-work from a single diagram or photo. For example, I originally believed that watch No.28 with equal segments would wind correctly. And it was only when I had simulated the motion of the wheels and barrel that I discovered it did not work.

So many of the ideas in this article were developed by animating the stop-work, studying the "video" and then explaining that behaviour.

Because I do not have any software to create animations, I have used Photoshop. The animations are crude, but they clearly show the behaviour of the different stop-work designs.

The animations were created by making a multi-layer diagram of the stop-work in Photoshop. Then the appropriate parts of the diagram were rotated and saved as a new file. This was repeated as many times as necessary, often about 100 times. After which, the files were loaded into a Photoshop stack, converted into frames and saved as an animation.

Because of small variations that caused some parts not to be strictly circular, rotating parts usually caused the center to move away from its correct position. Although I have attempted to nudge these parts back into their correct positions, there are often small movements in the animations caused by centers moving.

9: References

Antiquorum, 1991, *The Art of Breguet*, Switzerland: Antiquorum.

Berthoud, Ferdinand and Auch, Jacob, 1763 and 1827, *How to Make a Verge Watch*, Australia: Richard Watkins. (Available from www.watkinsr.id.au).

Blakey, W, 1780, *The Art of Making Watch Mainsprings, Repeater Springs and Balance Springs*, Australia: Richard Watkins. (Available from www.watkinsr.id.au).

Breguet, C, ca 1964, *A.L. Breguet horologer*, England: E.L. Lee.
In "Origins" (page 235) I mention that there is a survey of inventions dated 1796 that states Breguet's perpetuelle watches "are the result of 25 years study and meditation." The full text of the account is given in this book, pages 4 - 5. As I noted in "Origins", I think it unlikely that Breguet did anything more than meditate between 1771 and about 1776.

Breguet, Emmanuel, 1997, *Breguet, Watchmakers Since 1775*, Paris: Alain de Gourcuff.

Crespe, Francois, 1804, *Essay on Repeater Watches*, Australia: Richard Watkins. (Available from www.watkinsr.id.au).

Dubois, Pierre, 1849, *Histoire de l'Horlogerie depuis son Origine jusqu'à nos jours*, Paris: Administration du Moyen Age et la Renaissance.

Camus, C.E.L. and Hawkins, J.I., 1837 (1752), *A Treatise on the Teeth of Wheels*, London: J. Hodson, J. Weale & G. Herbert. (Available from www.watkinsr.id.au).

Chapuis, A and Jaquet, E, 1956: *The History of the Self-winding Watch 1770-1931*, London: B. T. Batsford.

Daniels, George, 1975, *The Art of Breguet*, London: Sotheby Parke Bernet.

Hatton, Thomas, 1773, *An Introduction to the Mechanical Part of Clock and Watch Work*, London: Longman and Robinson.

Houriet, Ch, *The American Watchmaking Factories and their Tools*, 1893 and 2010, available from www.watkinsr.id.au.

Jaquet, E and Chapuis, A, 1953: *Technique and History of the Swiss Watch*, Switzerland: Urs Graf-Verlag.

Martin, Benjamin, 1759, *A New and Comprehensive System of Mathematical Institutions, Agreeable to the Present State of the Newtonian Mathesis*, Volume 1, London: W. Owen.

Paley, William, 1802, *Natural Theology or Evidences of the Existence and Attributes of the Deity*, London: R. Faulder.

Sabrier, Jean Claude, 2012, *The Self-Winding Watch, 18th - 21st Century*, Paris: Editions Cercle d'Art.

Salomons, D. L., 1921, *Breguet (1747-1823)*, London: The Author.

Watkins, Richard, 2011, *The Repeater,* Australia: Richard Watkins. (Available from http://www.watkinsr.id.au/repeater.html).

Watkins, Richard, 2016, *The Origins of Self-Winding Watches, 1773 - 1779*, second edition, Tasmania; Richard Watkins. (Available from www.watkinsr.id.au).

Vigniaux, 1788, *Practical Watchmaking*, Australia: Richard Watkins. (Available from www.watkinsr.id.au).

Weiss, Leonard, 1982, *Watch-Making in England, 1760-1820*, London: Robert Hale.